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Exp. Prove that $\frac{x}{a}+\frac{y}{b}=1$ touches the curve $y=b\,e^{-x/a}$ at the point where the curve crosses the axis of y.

Sol.:- The x-co-ordinate of the curve cuts at the point the y axis is 0.

In order to find the y co-ordinate of the point putting $x=0$ in the given curve

$$y=b\,e^{-x/a}=b\,e^{0}=b$$

point $(0,b)$

Given the curve $y=b\,e^{-x/a}$

Diff. w.r.t x

$$\frac{dy}{dx}=b\cdot e^{-x/a}\left(-\frac{1}{a}\right)=-\frac{b}{a}e^{-x/a}$$

$$\left(\frac{dy}{dx}\right)_{\substack{x=0\\y=b}}=-\frac{b}{a}\cdot x^{0}=-\frac{b}{a}$$

Hence the required Equation of the tangent at the point $(0,b)$ is

$$(y-b)=-\frac{b}{a}(x-0)$$

$$y-b=-\frac{b}{a}x$$

$$ay-ab=-bx \Rightarrow bx+ay=ab$$

$$\Rightarrow \frac{x}{a}+\frac{y}{b}=1$$