Ex The n^{th} derivative (differential coefficient) of $(ax+b)^m$, where m is +ve integer $\geq n$.

Solution:- Let $y = (ax+b)^m$ —— (1)

On differentiating both sides w.r.t. x successively, we get.

$$y_1 = \frac{dy}{dx} = ma(ax+b)^{m-1}$$

$$y_2 = \frac{d^2y}{dx^2} = m(m-1)a^2(ax+b)^{m-2}$$

$$y_3 = \frac{d^3y}{dx^3} = m(m-1)(m-2) \cdot a^3 \cdot (ax+b)^{m-3}$$

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$$y_n = \frac{d^ny}{dx^n} = m(m-1)(m-2) \cdots [m-(n-1)] a^n (ax+b)^{m-n} \quad \text{—— (2)}$$

Again, differentiating Eqn (2)

$$y_{n+1} = \frac{d^{n+1}y}{dx^{n+1}} = m(m-1)(m-2) \cdots [m-(n-1)](m-n)a^{n+1}(ax+b)^{m-(n+1)}$$

Since, y_{n+1} is of the same form as y_n in Eqn (2)

If Eqn (2) is true for a particular value of n, then Eqn (2) is true for next higher value of n.

Eqn (2) is true for $n = 1, 2, 3, \cdots n$.

Therefore, by mathematical induction, Eqⁿ (2) is true for every positive integer n.

From equation (2), we can write y_n as follows

$$y_n = \frac{m(m-1)(m-2)\cdots(m-(n-1))(m-n)(m-n-1)\cdots 3.2.1}{(m-n)(m-n-1)\cdots 3.2.1} \times a^n (ax+b)^{m-n}$$

i.e $$\boxed{y_n = \frac{m!}{(m-n)!} a^n (ax+b)^{m-n}}$$

$$\Rightarrow y = (ax+b)^m \Rightarrow y_n = a^n (ax+b)^{m-n} \cdot \frac{\lfloor m}{\lfloor m-n}$$

put $a=1$ & $b=0$

$$y = x^m \Rightarrow y_n = \frac{\lfloor m}{\lfloor m-n} \cdot 1^n (1.x+0)^{m-n}$$

$$\boxed{y_n = \frac{\lfloor m}{\lfloor m-n} \cdot x^{m-n}}$$