A quasihomogeneous equation may be reduced to a homogeneous one by introducing a new variable z. i.e.

$$y = z^{\alpha}$$

where α to be chosen such that all the terms in the equation are of the same degree.

<u>Exp.</u> Solve the equation

$$2xy'(x-y^2)+y^3=0$$

<u>Solution</u> Put $y = z^{\alpha}$ we have

$$y' = \alpha z^{\alpha-1} \cdot z'$$

in equation, we get

$$2x\alpha z^{\alpha-1} z'(x - z^{2\alpha}) + z^{3\alpha} = 0$$

The first term $2x^2\alpha z^{\alpha-1}z'$ is of degree $(\alpha-1)+2$ $= \alpha+1$, the second term $-2x\alpha z^{3\alpha-1}\cdot z'$ is of degree $(3\alpha-1+1) = 3\alpha$ and the last term $z^{3\alpha}$ is of degree 3α. All the terms are of the same degree if $\alpha+1 = 3\alpha$ or $\alpha = 1/2$

The substitution $y = z^{1/2}$, we have

$$x \cdot z^{-1/2} z'(x-z) + z^{3/2} = 0$$

$$z' = \frac{z^2}{x(z-x)}, \quad x \neq 0$$

This equation being homogeneous, we get and set $z = ux$ and have

$\Rightarrow z = y^2$
$ux = y^2$
$u = y^2/x$

$$\frac{dz}{dx} = x\frac{du}{dx} + u = \frac{u^2}{u-1}$$

$$x\frac{du}{dx} = \frac{u^2}{u-1} - u$$

$$x\frac{du}{dx} = \frac{u^2 - u^2 + u}{u-1}$$

$$x\frac{du}{dx} = \frac{u}{u-1}$$

$(\because u \neq 0)$

$$\frac{u-1}{u}\,du = \frac{dx}{x}$$

$$du - \frac{du}{u} = \frac{dx}{x}$$

Integrating both sides

$$\int du - \int\frac{du}{u} = \int\frac{dx}{x} + \log c$$

$$u - \log u = \log x + \log c$$

Putting y^2/x for u we have the general solution of the given equation.

$$y^2 = x\ln|cy^2|$$