Exp. We consider once again the differential equation $\frac{dy}{dx} = 1+y^2$ given in previous example of fourth-order Runge-kutta method with the same condition and we wish to compute $y(0.8)$.

Solution. Given in Example of Runge kutta fourth order formula computed the values of $y(0.6)$, $y(0.4)$ and $y(0.2)$ by the fourth-order Runge kutta formula with using now

$$y(0.6) = y_0 = \cancel{.6841}\ .6841$$

$$y(0.4) = y_{-1} = 0.4228$$

$$y(0.2) = y_{-2} = 0.2027$$

$$y(0) = y_{-3} = 0$$

we obtain.

$$y_1^p = y_0 + \frac{h}{24}(55 f_0 - 59 f_{-1} + 37 f_{-2} - 9 f_{-3}]$$

$$f_0 = 1 + y_0^2 = 1 + (.6841)^2 = 1.46799 = 1.4680$$

$$f_{-1} = 1 + y_{-1}^2 = 1 + (0.4228)^2 = 1.1787$$

$$f_{-2} = 1 + y_{-2}^2 = 1 + (.2027)^2 = 1.041$$

$$f_{-3} = 1 + y_{-3}^2 = 1 + (0)^2 = 1.0$$

$$y_1^p = .6841 + \frac{0.2}{24}[55 \times 1.4680 - 59 \times 1.1787 + 37 \times 1.041 - 9 \times 1]$$

$$\cancel{0.2\,[1.698375]}$$

$$y_1^p = .6841 + \cancel{.339275} = 1.023375$$

Using the predicted value $y_1^p = 1.0233$ in corrector value formula

$$y_{n+1}^{p} = y_n + \frac{h}{24}\left[9f_{n+1}^{p} + 19f_n - 5f_{n-1} + f_{n-2}\right]$$

putting $n=0$

$$y_1^{c} = y_0 + \frac{h}{24}\left[9f_1^{p} + 19f_0 - 5f_{-1} + f_{-2}\right]$$

$$y_1^{c} = .6841 + \frac{0.2}{24}\left[9\times(1+(1.0233)^2) + 19(1+(.6841)^2) - 5\times(1+(.4228)^2) + (1+(.2027)^2)\right]$$

$$= .6841 + \frac{0.2}{24}\left[10.4257 + 27.8919 - 5.8938 + 1.0411\right]$$

$$= .6841 + .2(1.7277)$$

$$= .6841 + .3455$$

$= 1.02964$ which is correct to four decimal

$$y^c(0.8) = 1.0296$$