Exp. Using Muller's method find the root of the equation $f(x) = x^3 - x - 1 = 0$ with the initial approximations $x_{i-2} = 0$, $x_{i-1} = 1$, $x_i = 2$

Solution We have

$$y_{i-2} = -1, \quad y_{i-1} = -1, \quad y_i = 5$$

Also, $h_i = 1$, $h_{i-1} = 1$

$\Delta_i = 6 \quad \Delta_{i-1} = 0$

$\because h_i = x_i - x_{i-1}$

$h_{i-1} = x_{i-1} - x_{i-2}$

$\Delta_i = y_i - y_{i-1}$

$\Delta_{i-1} = y_{i-1} - y_{i-2}$

Hence, the equations gives A & B as

$$A = \frac{1}{(h_{i-1} + h_i)}\left(\frac{\Delta_i}{h_i} - \frac{\Delta_{i-1}}{h_{i-1}}\right) \quad \text{--- (I)}$$

$$A = \frac{1}{(1+1)}\left(\frac{6}{1} - \frac{0}{1}\right) = \frac{6}{2} = 3$$

$$B = \frac{\Delta_i}{h_i} + A h_i = \frac{6}{1} + 3 \times 1 = 9 \quad \text{--- (II)}$$

Then $\sqrt{B^2 - 4Ay_i} = \sqrt{(9)^2 - 4 \times 3 \times 5} = \sqrt{81 - 60}$

$= \sqrt{21}$

Therefore The equation

$$x_{i+1} = x_i - \frac{2y_i}{B \pm \sqrt{B^2 - 4Ay_i}} \quad \text{--- (III)}$$

gives

$$x_{i+1} = 2 - \frac{2(5)}{9 \pm \sqrt{21}}$$, since the sign of B is positive

$$x_{i+1} = 1.26376$$

Error in the above result $= \left|\frac{1.26376 - 2}{1.26376}\right| 100 = 58\%$

For the second approximation, we take

$x_{i-2} = 1$, $x_{i-1} = 2$, $x_i = 1.26376$

and corresponding values of y are

$y_{i-2} = -1$, $y_{i-1} = 5$, $y_i = -0.24542$

The computed values of A and B are

$A = 4.26375$ using Eqn (i)

$B = 3.98546$ using Eqn (ii)

Then $x_{i+1} = 1.32174$ using Eqn (iii)

and the error in the above result $= 4.39\%$

For the third approximation, we take

$x_{i-2} = 2$, $x_{i-1} = 1.26376$, $x_i = 1.32174$

$y_{i-2} = 5$, $y_{i-1} = -0.24542$, $y_i = -0.01266$

Then $A = 4.58544$, $B = 4.28035$ and

$x_{i+1} = 1.32469$

Error in the result $= 0.22\%$

For the next approximation, we have

$x_{i-2} = 1.26376$, $x_{i-1} = 1.32174$, $x_i = 1.32469$

Thus these values give

$A = 3.87920$, $B = 4.26229$ and $x_{i+1} = 1.32472$

The error in this result $= 0.002\%$

Hence, the required root is $\underline{1.3247}$, correct to 4 decimal places.