Linear Differential Equation

Exp. Solve $(D-2)^2 = 8(e^{2x}+\sin 2x+x^2)$

Solution - Given diff. Eqn.

$$(D-2)^2 = 8(e^{2x}+\sin 2x+x^2) \quad \text{---}(1)$$

(i) To find C.F (Complementary function)

The auxilary Eqn is $(D-2)^2 = 0$

$$\Rightarrow D = 2, 2 \quad (\text{root equal})$$

then

$$C.F = (c_1+c_2x)e^{2x} \quad \text{---}(ii)$$

(ii) To find P.I (Particular integral)

$$P.I = 8\Big[\underbrace{\frac{1}{(D-2)^2}e^{2x}}_{I} + \underbrace{\frac{1}{(D-2)^2}\sin 2x}_{II} + \underbrace{\frac{1}{(D-2)^2}x^2}_{III}\Big]$$

Now I $\frac{1}{(D-2)^2}e^{2x} = x^2\frac{1}{2\cdot(1)}e^{2x}$, put $D=2$ ∵ $f(D)=0$ & put $D=2$ $f'(D)=0$ but put $D=2$ $f''(D)\neq 0$

II $\frac{1}{(D-2)^2}\sin 2x = \frac{1}{D^2-4D+4}\sin 2x$ put $D^2 = -2^2$, $D^2=-4$

$$= \frac{1}{-2^2-4D+4}\sin 2x$$

$$= \frac{1}{-4D}\sin 2x = -\frac{1}{4}D^{-1}(\sin 2x)$$

$$= -\frac{1}{4}\int \sin 2x\,dx$$

$$\frac{1}{(D-2)^2}\sin 2x = -\frac{1}{4}\left(-\frac{\cos 2x}{2}\right) = \frac{1}{8}\cos 2x$$

III $\frac{1}{(D-2)^2}\cdot x^2 = \frac{1}{4}\left(1-\frac{D}{2}\right)^{-2}x^2$

$$= \frac{1}{4}\left[1+2\cdot\frac{D}{2}+3\frac{D^2}{4}+4\frac{D^3}{8}+\cdots\right]x^2$$

$$\Rightarrow \frac{1}{(D-2)^2} \cdot x^2 = \frac{1}{4}\left[x^2 + \frac{4}{2}\frac{d}{dx} x^2 + \frac{3}{4}\frac{d^2}{dx^2}x^2 + \cdots\right]$$

$$= \frac{1}{4}\left[x^2 + 2x + \frac{3}{4}\cdot 2 \cdot 1\right]$$

$$= \frac{1}{4}\left[x^2 + 2x + \frac{3}{2}\right]$$

Then

$$P.I = 8\left[\frac{1}{2}x^2 e^{2x} + \frac{1}{8}\cos 2x + \frac{1}{4}\left(x^2 + 2x + \frac{3}{2}\right)\right]$$

$$P.I = \left[4x^2 e^{2x} + \cos 2x + 2x^2 + 4x + 3\right]$$

Hence general solution is

$$y = C.F + P.I$$

$$y = (c_1 + c_2 x)e^{2x} + 4x^2 e^{2x} + \cos 2x + 2x^2 + 4x + 3$$