Exp. Find a quadratic factor of the polynomial

$$f(x) = x^3 - x - 1$$

Solution We have

$$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$$

then

$$a_3 = 1, \quad a_2 = 0, \quad a_1 = -1 \quad \& \quad a_0 = -1$$

Let $r_0 = s_0 = 1.0$

Then.

$$b_3 = a_3 \qquad \Rightarrow b_3 = 1$$

$$b_2 = a_2 - rb_3 = a_2 - ra_3 \quad \text{—— (i)} \Rightarrow b_2 = -1$$

$$b_1 = a_1 - rb_2 - sb_3 = a_1 - r(a_2 - ra_3) - sa_3$$

$$b_1 = a_1 - ra_2 + r^2a_3 - sa_3 \quad \text{—— (ii)}$$

$$\Rightarrow b_1 = -1 - 0 + 1\cdot1 - 1\cdot1$$

$$b_1 = -1$$

$$b_0 = a_0 - sb_2 = a_0 - s(a_2 - ra_3)$$

$$b_0 = a_0 - sa_2 + sra_3 \quad \text{—— (iii)}$$

$$b_0 = -1 - 1.0 + 1.1(1) = 0$$

Therefore,

$$\frac{\partial b_0}{\partial r} = sa_3, \quad \frac{\partial b_0}{\partial s} = -a_2 + ra_3 \qquad \text{from (iii)}$$

$$\frac{\partial b_1}{\partial r} = -a_2 + 2ra_3, \quad \frac{\partial b_1}{\partial s} = -a_3 \qquad \text{from (ii)}$$

Then by Eqn^s.

$$b_0(r,s) = b_0(r_0,s_0) + \frac{\partial b_0}{\partial r}\Delta r_0 + \frac{\partial b_0}{\partial s}\Delta s_0 = 0$$

$$b_1(r,s) = b_1(r_0,s_0) + \frac{\partial b_1}{\partial r}\Delta r_0 + \frac{\partial b_1}{\partial s}\Delta s_0 = 0$$

give

$$\Delta r_0 + \Delta s_0 = 0$$

$$2\Delta r_0 - \Delta s_0 = 0$$

Hence, $\Delta r_0 = \frac{1}{3} = 0.3333$

and $\Delta s_0 = -\frac{1}{3} = -0.3333$

Then

$$r_1 = r_0 + \Delta r_0 = 1 + \frac{1}{3} = 1.3333$$

$$s_1 = s_0 + \Delta s_0 = 1 - \frac{1}{3} = 0.6667$$

for the second approximation, we get

$$r_0 = 1.3333 \text{ and } s_0 = 0.6667$$

Then $b_0 = -0.1111$ and $b_1 = 0.1110$

$$\frac{\partial b_0}{\partial r} = 0.6667, \quad \frac{\partial b_0}{\partial s} = 1.3333$$

$$\frac{\partial b_1}{\partial r} = 2.6666, \quad \frac{\partial b_1}{\partial s} = -1$$

with these value, we obtain

$\Delta r_0 = -0.00874$ and $\Delta s_0 = 0.0877$

which give $r_2 = 1.3246$ and $s_2 = 0.7544$

both of which are correct to three decimal places.