Exp The mapping $T : V(R^3) \to V(R^2)$ defined as $T(x_1, x_2, x_3) = (x_1, x_2)$ is homomorphism (Linear Transformation) of $V(R^3)$ onto $V_2(R^2)$ or $V_3(R)$ onto $V_2(R)$

Solution Let $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$ belong to $V_3(R)$ i.e. $x, y \in V_3(R)$

Also α & $\beta \in (R)$ and condition $T(x_1, x_2, x_3) = (x_1, x_2)$

We have Linear Transformation —①

Now

$$T(\alpha x + \beta y) = \alpha T(x) + \beta T(y)$$

$$\because T(\alpha x + \beta y) = T[\alpha(x_1, x_2, x_3) + \beta(y_1, y_2, y_3)]$$

$$= T[(\alpha x_1, \alpha x_2, \alpha x_3) + (\beta y_1, \beta y_2, \beta y_3)]$$

$$= T(\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2, \alpha x_3 + \beta y_3)$$

$$= (\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2) \text{ by Cond ①}$$

$$\therefore T(\alpha x + \beta y) = (\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2) \text{ —②}$$

Now

$$\alpha T(x) + \beta T(y) = \alpha T(x_1, x_2, x_3) + \beta T(y_1, y_2, y_3)$$

$$= \alpha(x_1, x_2) + \beta(y_1, y_2)$$

— by Cond. ①

$$= (\alpha x_1, \alpha x_2) + (\beta y_1, \beta y_2)$$

$$\alpha T(x) + \beta T(y) = (\alpha x_1, \alpha x_2) + (\beta y_1, \beta y_2)$$

$$\alpha T(x) + \beta T(y) = (\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2) \quad \text{--- (3)}$$

from (2) & (3)

$$T(\alpha x + \beta y) = \alpha T(x) + \beta T(y)$$

$\because$ Hence $T: V(R^3) \to V(R^2)$ is Linear Transformation or homomorphism for condition $T(x_1, x_2, x_3) = (x_1, x_3)$