Exp. Using Graeffe's method, find the real roots of the equation : $x^3-6x^2+11x-6=0$

Sol. Let

$$f(x)=x^3-6x^2+11x-6=0 \quad \text{—— (1)}$$

then

$$f(-x)=-x^3-6x^2-11x-6=0$$

therefore,

$$f(x)f(-x)=(-1)^3(x^6-14x^4+49x^2-36)$$

Let $\phi(z)=z^3-14z^2+49z-36$, where $z=x^2$

Hence roots of $f(x)=0$ are given by

$$\sqrt{\frac{36}{49}}=\frac{6}{7}=0.857,\ \sqrt{\frac{49}{14}}=1.871 \text{ and } \sqrt{14}=3.742$$

Now, $\phi(-z)=-z^3-14z^2-49z-36$

Therefore,

$$\phi(z)\phi(-z)=(-1)^3(z^6-98z^4+1393z^2-1296)$$

Setting $\phi(u)=u^3-98u^2+1393u-1296$, where $u=z^2$, we obtain the next approximation to the roots of $f(x)=0$

$$\left(\frac{1296}{1393}\right)^{1/4}=0.9822,\ \left(\frac{1393}{98}\right)^{1/4}=1.942$$

and $(98)^{1/4}=3.147$

So, that the approximations are covering to the actual root 1, 2 & 3 respectively.

We use $x_0=0.857$ and apply Newton's method

We obtain $x_1=0.857+\dfrac{0.3507}{2.919}=0.977.$