Exp. Solve the system of Equations

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16$$

by using Gauss Elimination and Gauss-Jordan methods.

Solution Write the system of Eqn. in this order

$$x + 4y + 9z = 16 \quad \text{—— (i)}$$
$$2x + y + z = 10 \quad \text{—— (ii)}$$
$$3x + 2y + 3z = 18 \quad \text{—— (iii)}$$

System of Eqs. write in Augmented matrix $(A : B)$ form

$$(A : B) = \begin{bmatrix} 1 & 4 & 9 & : & 16 \\ 2 & 1 & 1 & : & 10 \\ 3 & 2 & 3 & : & 18 \end{bmatrix}$$

(1) Using Gauss Elimination method.

→ Augmented Matrix makes in Upper-Triangular matrix form for Gauss Elimination method

$$\begin{array}{cc|c} \checkmark & \checkmark & \checkmark \\ 0 & \checkmark & \checkmark \\ 0 & 0 & \checkmark \end{array}$$

$$(A : B) = \begin{bmatrix} 1 & 4 & 9 & : & 16 \\ 2 & 1 & 1 & : & 10 \\ 3 & 2 & 3 & : & 18 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

then.

$$= \left[\begin{array}{ccc:c} 1 & 4 & 9 & 16 \\ 0 & -7 & -17 & -22 \\ 0 & -10 & -24 & -30 \end{array}\right] \quad R_3 \to 7R_3 - 10R_2$$

$$= \left[\begin{array}{ccc:c} 1 & 4 & 9 & 16 \\ 0 & -7 & -17 & -22 \\ 0 & 0 & 2 & 10 \end{array}\right]$$

$$x + 4y + 9z = 16 \longrightarrow \text{(iv)}$$

$$-7y - 17z = -22 \longrightarrow \text{(v)}$$

$$2z = 10 \longrightarrow \text{(vi)} \Rightarrow z = 5$$

from Back substitution.

from (v) $y = -9$

from (iv) $x = 7$

Hence the solution of system Eq.

$x = 7,\ y = -9\ \&\ z = 5$

(ii) using Gauss-Jordan method.

Augmented matrix of Gauss Elimination method makes diagonal matrix form.

✓	0	0
0	✓	0
0	0	✓

Using augmented matrix from gauss Elimination method.

$$(A:B) = \begin{bmatrix} 1 & 4 & 9 & : & 16 \\ 0 & -7 & -17 & : & -22 \\ 0 & -10 & -24 & : & -30 \end{bmatrix}$$

$R_1 \to 7R_1 + 4R_2$

$R_3 \to 7R_3 - 10R_2$

$$= \begin{bmatrix} 7 & 0 & -5 & : & 24 \\ 0 & -7 & -17 & : & -22 \\ 0 & 0 & 2 & : & 10 \end{bmatrix}$$

$R_1 \to 2R_1 + 5R_3$

$R_2 \to 7R_3 + 2R_2$

$$= \begin{bmatrix} 14 & 0 & 0 & : & 98 \\ 0 & -14 & -17 & : & 126 \\ 0 & 0 & 2 & : & 10 \end{bmatrix}$$

$14x = 98 \Rightarrow x = 7$

$-14y = 126 \Rightarrow y = -9$

$2z = 10 \Rightarrow z = 5$

Hence the solution of Eqn are

$x = 7, \; y = -9 \; \& \; z = 5$