

Exp. Solve the following by Euler's Modified method

$$\frac{dy}{dx} = f(x, y) = \log(x+y), \quad y(0) = 2$$

at  $x = 1.2$  and  $1.4$  with  $h = 0.2$ .

Solution. Given that  $\frac{dy}{dx} = \log(x+y)$   
 $x = 0.0 \rightarrow y(0) = 2$  and  $h = 0.2$

Then the various calculations are arranged as follows:-

$x$	$\log(x+y) = \frac{dy}{dx}$	Mean slope	$dy + h \cdot \text{Mean slope} = y$
0.0	$\log(0+2) = 0.301$	—	$2 + 0.2(0.301) = 2.0602$
0.2	$\log(0.2 + 2.0602) = 0.3541$	$\frac{1}{2}(0.301 + 0.3542)$	$2 + 0.2(0.3276) = 2.0656$
0.2	$\log(0.2 + 2.0656) = 0.3552$	$\frac{1}{2}(0.301 + 0.3552)$	$2 + 0.2(0.3281) = 2.0656$
0.2	0.3552	—	$2.0656 + 0.2(0.3552) = 2.1366$
0.4	$\log(0.4 + 2.1366) = 0.4042$	$\frac{1}{2}(0.3552 + 0.4042)$	$2.0656 + 0.2(0.3797) = 2.1415$
0.4	$\log(0.4 + 2.1415) = 0.4051$	$\frac{1}{2}(0.3552 + 0.4051)$	$2.0656 + 0.2(0.3801) = 2.1416$
0.4	0.4051	—	$2.1416 + 0.2(0.4051) = 2.2226$
0.6	$\log(0.6 + 2.2226) = 0.4536$	$\frac{1}{2}(0.4051 + 0.4536)$	$2.1416 + 0.2(0.4294) = 2.2272$
0.6	$\log(0.6 + 2.2272) = 0.4514$	$\frac{1}{2}(0.4051 + 0.4514)$	$2.1416 + 0.2(0.4281) = 2.2272$
0.6	0.4514	—	$2.2272 + 0.2(0.4514) = 2.3175$
0.8	$\log(0.8 + 2.3175) = 0.4932$	$\frac{1}{2}(0.4514 + 0.4932)$	$2.2272 + 0.2(0.4723) = 2.3217$
0.8	$\log(0.8 + 2.3217) = 0.4943$	$\frac{1}{2}(0.4514 + 0.4943)$	$2.2272 + 0.2(0.4727) = 2.3217$
0.8	0.4943	—	$2.3217 + 0.2(0.4943) = 2.4206$
1.0	$\log(1.0 + 2.4206) = 0.5341$	$\frac{1}{2}(0.4943 + 0.5341)$	$2.3217 + 0.2(0.5142) = 2.4245$
1.0	$\log(1.0 + 2.4245) = 0.5346$	$\frac{1}{2}(0.4943 + 0.5346)$	$2.3217 + 0.2(0.5144) = 2.4245$
1.0	0.5346	—	$2.4245 + 0.2(0.5346) = 2.5314$
1.2	$\log(1.2 + 2.5314) = 0.5719$	$\frac{1}{2}(0.5346 + 0.5719)$	$2.4245 + 0.2(0.5532) = 2.5357$
1.2	$\log(1.2 + 2.5357) = 0.5727$	$\frac{1}{2}(0.5346 + 0.5727)$	$2.4245 + 0.2(0.5534) = 2.5357$

Hence value of  $y = 2.5357$  at  $x = 1.2$  and we also

$x$	$\log(x+y) = y'$	mean slope	$ddy + 2(\text{mean slope}) = ny$
1.2	.5723	—	$2.5351 + 2(.5723)$ $= 2.6496$
1.4	$\log(1.4 + 2.6496) = .6074$	$\frac{1}{2} \begin{pmatrix} .5723 \\ 2.6496 \end{pmatrix}$	$2.5351 + 2(.5898)$ $= 2.6531$
1.4	$\log(1.4 + 2.6531) = .6078$	$\frac{1}{2} \begin{pmatrix} .5723 \\ 2.6078 \end{pmatrix}$	$2.5351 + 2(.5700)$ $= 2.6531$

at  $x = 1.4$  the value  $y$  is 2.6531