Exp. We consider the initial value problem $\frac{dy}{dx} = 3x + y/2$ with the initial condition $y(0) = 1$ when $x = 0$ then find the value $y(0.2)$ by using differen methods, Euler's, modified Euler's and Runge-kutta fourth-order methods. The exact value of $y(0.2)$ is being 1.16722193.

Solution- given that $f(x,y) = \frac{dy}{dx} = 3x + y/2$

$y(0) = 1$ when $x = 0$ and $h = 0.05$

(1) By using Euler's method.

x	y	$f(x,y) = 3x + y/2$	old y + $hf(x,y)$ = new y
0.0	1.0	$3(0) + \frac{1}{2} = 0.5$	$1.0 + .05(0.5) = 1.025$
0.05	1.025	$3(0.05) + \frac{1.025}{2} = .6625$	$1.025 + .05(.6625) = 1.058$
0.15	1.058	$3(.15) + \frac{1.058}{2} = .9790$	$1.058 + .05(.979) = 1.107$
0.20	1.107	$3(.2) + \frac{1.107}{2} = 1.153$	$1.107 + 0.05(1.153) = 1.165$

$y(0.2) = 1.107$

(2) By using Modified Euler Method

x	$dy/dx = f(x,y) = 3x + y/2$	mean slope	old y + $h \cdot$ mean slope = y
0.0	$3(0.0) + \frac{1}{2} = 0.5$	—	$1 + 0.05(.5) = 1.025$
0.05	$3(.05) + \frac{1.025}{2} = .6625$	.58125	$1 + 0.05(.581) = 1.029$
0.05	$3(0.05) + \frac{1.029}{2} = .6645$	.5825	$1 + 0.05(.5825) = 1.029$
0.05	.6645	—	$1.029 + 0.05(.6645) = 1.0623$
0.15	$3 \times (.10) + \frac{1.0623}{2} = .8311$	$\frac{1}{2}(.6645 + .8311) = 0.7478$	$1.029 + 0.05(.7478) = 1.0664$
0.15	$3(.10) + \frac{1.0664}{2} = .8332$	$\frac{1}{2}(.6645 + .8332) = .7488$	$1.029 + .05(.7488) = 1.0664$

Modified Euler's Method

x	$f(x,y) = 3x + y/2$	mean slope	old y + h (mean slope)
0.10	0.8372	—	$1.0665 + 0.05(0.8372) = 1.1081$
0.15	$3(.15) + \frac{1.1081}{2} = 1.004$	$\frac{1}{2}(0.8372 + 1.004) = .9186$	$1.0665 + 0.05(.9186) = 1.1123$
0.15	$3(.15) + \frac{1.1123}{2} = 1.006$	$\frac{1}{2}(0.8372 + 1.006) = 0.9197$	$1.0665 + 0.05(0.9197) = 1.1124$
0.15	1.006	—	$1.1124 + 0.05(1.006) = 1.1627$
0.20	$3(.2) + \frac{1.1627}{2} = 1.1814$	$\frac{1}{2}(1.006 + 1.1814) = 1.094$	$1.1124 + 0.05(1.094) = 1.1671$
0.20	$3(.2) + \frac{1.1671}{2} = 1.1836$	$\frac{1}{2}(1.006 + 1.1836) = 1.0948$	$1.1124 + 0.05(1.0948) = 1.16713$

$$y(0.2) = 1.1671$$

(III) By using Runge-Kutta Four order Method.

$$f(x,y) = 3x + y/2$$

$y(0) = 1$ when $x = 0$ and $h = 0.05$

$$f(x_0, y_0) = 3 \times 0 + \frac{1}{2} = 0.5$$

The fourth order Runge Kutta method

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where $k_1 = hf(x_0, y_0) = 0.05(0.5) = 0.025$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$k_2 = 0.05\left[3\left(0+\frac{0.05}{2}\right)+\frac{1}{2}\left(1+\frac{0.025}{2}\right)\right]$

$= 0.05[0.075 + .50625] = 0.0291$

$k_3 = hf\left(x_0+\frac{h}{2}, y_0+\frac{1}{2}k_2\right)$

$= 0.05\left[3\left(0+\frac{0.05}{2}\right)+\frac{1}{2}\left(1+\frac{0.0291}{2}\right)\right]$

$k_3 = 0.05[0.075 + .5073] = 0.02911$

$k_4 = hf(x_0+h, y_0+k_3)$

$= 0.05\left[3\left(0+\frac{0.05}{2}\right)+\frac{1}{2}\left(1+\frac{0.02911}{2}\right)\right]$

$= 0.05[0.1750 + .51735]$

$k_4 = 0.66455$

$y_1 = y(0.05) = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= 1 + \frac{1}{6}[0.025 + 2\times 0.0291 + 2\times 0.02911 + 0.66455]$

$= 1 + 0.13432$

$y(0.5) = 1.13432 = 1.1343$

Hence. To compute $y(0.1)$ we take $x_0 = 0.05$ and $h = 0.05$ and $y_0 = 1.1343$ from using fourth order Runge kutta method.

Repeating this method for calculating $y(0.10)$, $y(0.15)$ and $y(0.2)$.

The we find $y(0.2) = 1.672$.