Exact Equation-

A diff. Eq. $M(x,y)\,dx + N(x,y)\,dy = 0$ —(1)
is said to be exact if there exists a function $u(x,y)$
such that

$$du = \frac{\partial u}{\partial x}\cdot dx + \frac{\partial u}{\partial y}\,dy = M\,dx + N\,dy.$$

Exp. The diff. Eqn. $\frac{1}{y}\,dx - \frac{x}{y^2}\,dy = 0$

This equation is exact, since it may be written

$$d\left(\frac{x}{y}\right) = 0 \Rightarrow \frac{x}{y} = c$$

and other hand the same equation in the form

$$y\,dx - x\,dy = 0 \quad \text{is not exact.}$$

Equation (1) is exact if and only if

$$\frac{\partial M}{\partial y} \equiv \frac{\partial N}{\partial x} \quad \text{—— (2)}$$

The solution of the equation (1) $M(x,y)dx + N(x,y)dy = 0$
is

$$\int_a^x M(x,y)\,dx + \int_b^y N(a,y)\,dy = c \quad \text{—— (3)}$$

Where a, b are any convenient constants.

Exp. Solve the diff. Eqn.

$$2xy\,dx + (x^2 - y^2)\,dy = 0$$

Solution Given diff. Eqn compare with

$$M(x,y)dx + N(x,y)dy = 0$$

Thus $\Rightarrow$ $M(x,y) = 2xy \Rightarrow \frac{\partial(M(x,y))}{\partial y} = 2x\cdot 1 = 2x$

$$N(x,y) = x^2 - y^2 \Rightarrow \frac{\partial N(x,y)}{\partial x} = 2x - 0 = 2x$$

It is clear that $\frac{\partial M(x,y)}{\partial y} = 2x = \frac{\partial N(x,y)}{\partial x}$

Thus given equation is exact diff. Eqn.

Then solution of exact diff. Eqn.

$$\int_a^x 2xy\,dx + \int_b^y (a^2 - y^2)\,dy = c \quad \text{— (4)}$$

If we take $a=0$, $b=0$ several terms $\because N(a,y)=$ drop out and Eqn (4) gives

$$\int_0^x 2xy\,dx + \int_0^y (-y^2)\,dy = c$$

$$x^2y - \frac{y^3}{3} = c$$

$$3x^2y - y^3 = 3c$$

<u>Exercise-1</u> Solve the diff. Eqn.

$$\frac{y}{x}\,dx + (y^3 + \log x)\,dy = 0$$

<u>Exercise-2</u>- Solve the diff. Eqn.

$$2x + 4y + (2x - 2y)\,y' = 0$$

<u>Exercise-3</u> Solve the diff. Eqn.

$$\left(\frac{x}{\sin y} + 2\right)dx + \frac{(x^2+1)\cos y}{\cos 2y - 1}\,dy = 0$$