Exp. Solve the diff. Equation.

$$(x^3e^{(x+y)} + 3x^2e^{(x+y)} - x)dx + (x^3e^{(x+y)} + y)dy = 0$$

Solution - given diff. Equation compare with

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

$$\Rightarrow M(x,y) = x^3e^{x+y} + 3x^2e^{x+y} - x$$

$$N(x,y) = x^3e^{x+y} + y$$

This diff. Eqn. is exact then

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Thus $\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(x^3e^{x+y} + 3x^2e^{x+y} - x) = x^3 \cdot e^{x+y} + 3x^2e^{x+y}$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(x^3e^{x+y} + y) = x^3 \cdot e^{x+y} \cdot 1 + 3x^2e^{x+y}$$

Since $\frac{\partial M}{\partial y} = x^3e^{x+y} + 3x^2e^{x+y} = \frac{\partial N}{\partial x}$

So the diff. Eqn. is exact. We have

$$U(x,y) = \int(x^3e^{x+y} + 3x^2e^{x+y} - x)dx + f(y) \quad \text{---(I)}$$

Since the integration of $(x^3e^{x+y} + 3x^2e^{x+y})$ with respect to x is more complecated than $(x^3e^{x+y} + y)$ with respect to y, we use partial derivative

$$\frac{\partial u}{\partial y} = x^3e^{x+y} + y$$

as hold x constant. Integrating this with respect to y, gives $\quad U = x^3e^{x+y} + \frac{y^2}{2} + h(x) \quad$ ---(II)

$$\frac{\partial u}{\partial x} = 3x^2e^{x+y} + x^3e^{x+y} + h'(x) = 3x^2e^{x+y} + x^3e^{x+y} - x$$

$$\Rightarrow h'(x) = -x \quad \Rightarrow h(x) = \frac{-x^2}{2}$$

Then from (ii)

$$U(x,y) = x^3 e^{x+y} + \frac{y^2}{2} - \frac{x^2}{2} = c$$

This is solution.

Another Solution Method. (Mostly used)

$$\int M(x,y)\,dx + \int (\text{terms of } N \text{ not containing } x)\,dy$$
(y constant)

$$\int (x^3 e^{x+y} + 3x^2 e^{x+y} - x)\,dx + \int y\,dy = c$$

$$e^y \int x^3 e^x + e^y \int 3x^2 e^x - \int x\,dx + \int y\,dy = c$$

$$e^y \left[x^3 e^x - \int \cancel{3x^2 e^x dx}\right] + e^y \int \cancel{3x^2 e^x} dx - \frac{x^2}{2} + \frac{y^2}{2} = c$$

$$x^3 e^{x+y} + \frac{y^2}{2} - \frac{x^2}{2} = c$$

Ans

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P-21-ODE

Exercise Solve the diff. Eqn.

$$e^{-y}dx - (2y + x e^{-y})dy = 0$$

Hint:- Given diff. Eqn is exact i.e. $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Then there exists a function $U(x,y)$ such that

$$\frac{\partial U}{\partial x} = M = -e^{-y} \quad \text{and} \quad \frac{\partial U}{\partial y} = N = -(2y + x e^{-y})$$

and Integrating this Eqn w.r.t. x as y constant $U(x,y) = \int e^{-y} dx + h(y)$

$U(x,y) = e^{-y} x + h(y)$