RINGS

ELEMENTARY PROPERTIES OF A RING :-

To prove that for all a, b, c in a ring R

(i) $a0 = 0a = 0$

(ii) $a(-b) = -(ab) = (-a)b$

(iii) $(-a)(-b) = ab$

(iv) $a(b-c) = ab - ac$

(v) $(b-c)a = ba - ca$

Proof (i) $a0 = a(0+0)$ $(\because 0+0=0)$

$= a0 + a0$ (By left distributive law)

$0 + a0 = a0 + a0$ (By the property of identity) $(\because 0 + a0 = a0$ as $a0 \in R))$

or, $0 = a0$ (By right cancellation law $\because$ R is a group)

similarly, $0a = (0+0)a$ $(\because 0+0=0)$

$= 0a + 0a$ (By right distributive law)

or, $0a + 0 = 0a + 0a$ (By the property of identity)

or, $0 = 0a$ (By left cancellation law)

Hence $a0 = 0a = 0$

(ii) $a0 = 0$ [From (i)]

or, $a[(-b)+b] = 0$ (By the property of inverse in R)

or, $a(-b) + ab = 0$ (By left distributive law)

or, $a(-b) = -(ab)$ $(\because a+b=0 \Rightarrow a=-b)$

Similarly, $0b = 0$ [using (i)]

or, $[(-a)+a]b = 0$ (By the property of inverse in R)

or, $(-a)b + ab = 0$ (By right distributive law)

or, $(-a)b = -(ab)$ $(\because a+b=0 \Rightarrow a=-b)$

Hence $(-a)b = -(ab) = a(-b)$

(iii) $(-a)(-b) = (ab)$

Since we have $a(-b) = -(ab)$

$\therefore (-a)(-b) = -[(-a)b]$

$= -[-(ab)]$ $[\because (-a)b = -(ab)]$

$= ab$

Hence $(-a)(-b) = ab$

(iv) $a(b-c) = ab - ac$

Now $a(b-c) = a[b+(-c)]$ [by left distributive law]

$= ab + a(-c)$

$= ba - ca$ $[\because (-c)a = -(ca)]$

Hence $(b-c)a = ba - ca$.

(v) $(b-c)\,a = ba - ca$

Now $(b-c)\,a = [b + (-c)]\,a$

$= ba + (-c)a$ (by right distributive law)

$= ba - ca \quad [\because (-c)a = -(ca)]$

Hence $(b-c)a = ba - ca$.