Q(5) A particle moves along the arc of a cycloid in such a manner that the tangent to it rotates with a constant angular velocity. Show that the acceleration of the moving particle is constant in magnitude.

Let $s = 4a \sin\psi$ ——— (1)

be the cycloid. Then by question

$$\frac{d\psi}{dt} = \text{Constant} = k \text{ (say)} \quad \text{——— (2)}$$

Now the tangential acceleration $= \frac{d^2s}{dt^2}$

$$= \frac{d}{dt}\left(4a\cos\psi \frac{d\psi}{dt}\right)$$

$$= \frac{d}{dt}\left(4ak\cos\psi\right)$$

$$= -4ak^2 \sin\psi$$

The normal acceleration $= \frac{\left(\frac{ds}{dt}\right)^2}{\rho}$

$$= \frac{(4ak^2\cos\psi)^2}{\rho}$$

$$= \frac{4ak\cos\psi}{ds/d\psi}$$

$$= \frac{(4ak\cos\psi)^2}{4a\cos\psi}$$

$$(\text{Resultant acceleration})^2 = \left\{\frac{d^2s}{dt^2}\right\}^2 + \left\{\frac{\left(\frac{ds}{dt}\right)^2}{\rho}\right\}^2$$

$$\text{Resultant acceleration} = \sqrt{\{16a^2k^4\sin^2\psi + 16a^2k^4\cos^2\psi\}}$$

$$= \sqrt{16a^2k^4(\sin^2\psi + \cos^2\psi)}$$

$$= 4ak^2$$