Q. 2 A particle describes a rectangular hyperbola, the acceleration being directed from the centre. Show that the angle θ described about the centre in time t after leaving the vertex is given by $\tan\theta = \tanh(t\sqrt{\mu})$ where μ is the acceleration at distance unity.

Let the rectangular hyperbola be

$$x^2 - y^2 = a^2 \quad \text{——— (1)}$$

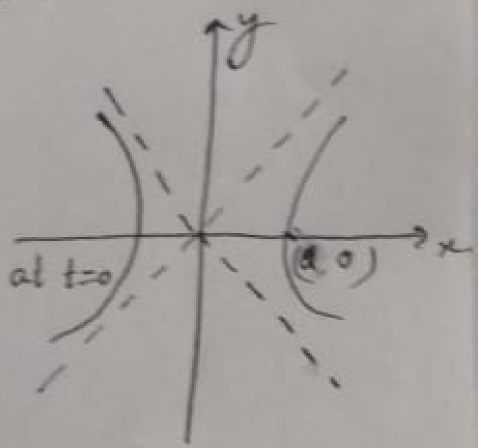
Then the equations of motion are

$$\frac{d^2x}{dt^2} = \mu x \quad \text{——— (2)}$$

and
$$\frac{d^2y}{dt^2} = \mu y \quad \text{——— (3)}$$

The initial conditions are

$$x = a, \quad y = 0, \quad \frac{dx}{dt} = 0$$



Integrating (2) we get

$$x = Ae^{\sqrt{\mu}t} + Be^{-\sqrt{\mu}t}$$

Integrating (3) we get

$$y = Ce^{\sqrt{\mu}t} + De^{-\sqrt{\mu}t}$$

The initial conditions give

$$a = A + B$$
$$0 = C + D$$
$$0 = \sqrt{\mu}\,(A - B)$$

or, $A = B = \frac{1}{2}a, \quad C = -D$

Hence $x = \frac{1}{2}a\left(e^{\sqrt{\mu}t} + e^{-\sqrt{\mu}t}\right)$

$= a\cosh(\sqrt{\mu}t)$

$y = c\left(e^{\sqrt{\mu}t} - e^{-\sqrt{\mu}t}\right)$

$= 2c\sinh(\sqrt{\mu}t)$

Substituting in ① we get $a^2\cosh^2(\mu t) - 4c^2\sinh^2(\mu t) = a^2$

or, $C = \frac{1}{2}a$

Hence $\tan\theta = \frac{y}{x} = \frac{a\sinh(\sqrt{\mu}t)}{a\cosh(\sqrt{\mu}t)}$

$= \tanh(\mu t)$