DYNAMICS

(1) (i) Deduce Newton's law of gravitation from Keplers law.

Keplers first law of planetory motion states that every pl describes an ellipse with the sun at one of the foci. This implies that the force of attraction exte by the sun on the planet is always directed towards the sun. Let us now find the law of for under which the plant describy an ellipse with sun at a focus.

We know that for any central orbit, the law of force is given by

$$P = h^2 u^2 \left[u + \frac{d^2u}{d\theta^2} \right] \quad \text{——} \; (1)$$

is used notation.

Now the equation to an ellipse refferd to a focus as t pole is,

$$r = \frac{l}{1+e\cos\theta}$$

This gives $u = \frac{1}{r} = \frac{1}{l} + \frac{e\cos\theta}{l}$

$$\therefore \frac{du}{d\theta} = \frac{-e\sin\theta}{l},$$

$$\frac{d^2u}{d\theta^2} = -\frac{e\cos\theta}{l}$$

Hence (1). gives

$$P = h^2 u^2 \left(\frac{1}{l}\right)$$

$$= \frac{h^2}{l} u^2$$

$$\therefore P = \frac{h^2}{l} \cdot \frac{1}{r^2}$$

Thus the force of attraction between the sun and the planet vanish inversely as the square of the distance between them

This is nothing but Newton's law of gravitation $\left[F = G \cdot \frac{Mm}{r^2} \propto \frac{1}{r^2}\right]$

Thus Kepler's 1st law $\Rightarrow$ Newton's law of gravitation