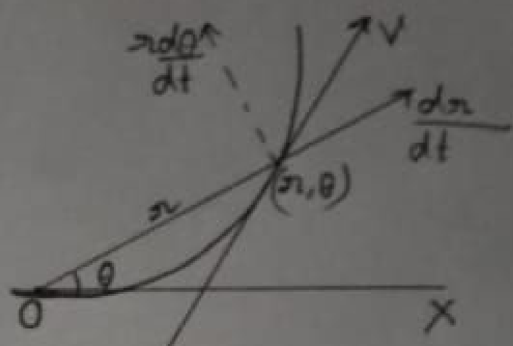
Q. (4) In a central orbit prove that

$$v^2 = h^2\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right]$$

Sol:- In a central orbit we have

$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = 0$$



This gives

$$r^2\frac{d\theta}{dt} = \text{Constant} = h \text{ (say)} \quad \text{—— (1)}$$

If v be the velocity at (r, θ) we have

$$v^2 = \left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\theta}{dt}\right)^2$$

$$= \left(\frac{dr}{d\theta}\frac{d\theta}{dt}\right)^2 + \left(r\frac{d\theta}{dt}\right)^2$$

$$= \left(\frac{d\theta}{dt}\right)^2\left[\left(\frac{dr}{d\theta}\right)^2 + r^2\right]$$

$$= \frac{h^2}{r^4}\left[\left(\frac{dr}{d\theta}\right)^2 + r^2\right] \quad \text{by (1)}$$

$$= h^2u^4\left[\left(-\frac{1}{u^2}\frac{du}{d\theta}\right)^2 + \frac{1}{u^2}\right] \quad \text{where } r=$$

$$= h^2\left[\left(\frac{du}{d\theta}\right)^2 + u^2\right]$$

Proved