DYNAMICS

Q.8 Obtain the result $v^2 = h^2\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right]$

$= \int \frac{2P\,du}{u^2}$ in case of Central orbits.

In case of Central orbit.

$$\frac{P}{u^2} = h^2\left[u + \frac{d^2u}{d\theta^2}\right]$$

or, $$\frac{2P}{u^2}\frac{du}{d\theta} = h^2\left[2u \cdot \frac{du}{d\theta} + 2\frac{du}{d\theta}\frac{d}{d\theta}\left(\frac{du}{d\theta}\right)\right]$$

Integrating w.r.t. θ we get

$$\int \frac{2P\,du}{u^2} = h^2\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right]$$

$$= v^2 \qquad [\text{from Q.4}]$$

<u>Note</u> :- If we are asked to find the velocity at any point of the path after (2) we write:—

Now we have seen that the path is a parabola if $C = 0$, an ellipse if $-\frac{2\mu}{C} = 2a$ and a hyperbola if $\frac{2\mu}{C} = 2a$, where $2a$ is the transverse axis of the ellipse or the hyperbola.

Hence from (2) we get

$$v = \begin{cases} \frac{\sqrt{2\mu}}{r} & \text{if the path is a parabola.} \\ \sqrt{\mu\left(\frac{2}{r} \mp \frac{1}{a}\right)} & \text{according as the path is an ellipse or a hyperbola.} \end{cases}$$