DYNAMICS

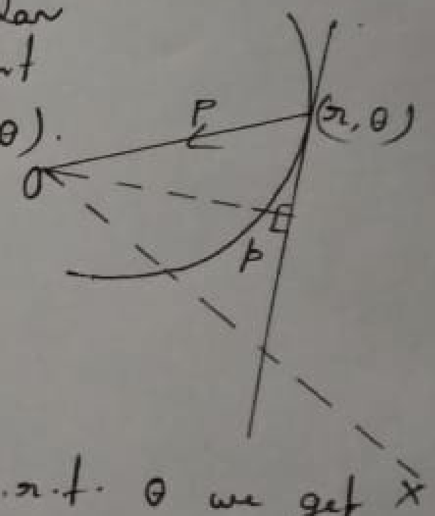
Q(3) Obtain the pedal equation of the path of a particle moving in the central field of force.

[OR, A particle moves in a plane with acceleration always towards a fixed point in the plane. Find the p-r equation of the path.]

sol. First we find the differential equation of the path of a particle moving in a central field of force in reciprocal polar coordinates. Then

add :-

Let p be perpendicular from O to the tangent to the curve at (r, θ). then from Differential Calculus,



we know that

$$\frac{1}{p^2} = u^2 + \left(\frac{du}{d\theta}\right)^2$$

Differentiating this w.r.t. θ we get

$$\left(-\frac{2}{p^3}\right)\frac{dp}{d\theta} = 2u\frac{du}{d\theta} + 2\left(\frac{du}{d\theta}\right)\cdot\frac{d^2u}{d\theta^2}$$

or, $$\left(\frac{-2}{p^3}\right)\frac{dp}{d\theta} = 2\frac{du}{d\theta}\left(u + \frac{d^2u}{d\theta^2}\right)$$

or, $$\left(\frac{-\cancel{2}}{p^3}\right)\frac{dp}{d\theta} = \cancel{2}\frac{du}{d\theta}\cdot\frac{P}{h^2u^2} \quad \text{by (3)}$$

$$\therefore -\frac{1}{p^3}\frac{dp}{dr}\cancel{\frac{dr}{d\theta}} = \frac{du}{dr}\cancel{\frac{dr}{d\theta}}\frac{P}{h^2u^2}$$

or, $$-\frac{1}{p^3}\frac{dp}{dr} = \frac{d}{dr}\left(\frac{1}{r}\right)\frac{P}{h^2}r^2$$

or, $$-\frac{1}{p^3}\frac{dp}{dr} = -\frac{1}{r^2}\cdot\frac{P}{h^2}r^2$$

or, $$-\frac{1}{p^3}\frac{dp}{dr} = -\frac{1}{r^2}\cdot\frac{P}{h^2}r^2$$

or, $$\frac{h^2}{p^3}\frac{dp}{dr} = P$$

This is the required differential equation.