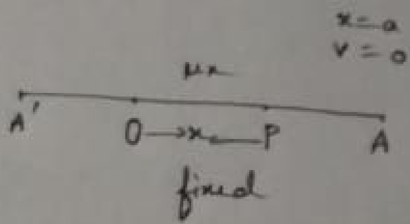
Equation (a) gives the velocity at point P.

Motion of a particle under Simple Harmonic motion :—

Let O is a fixed point then we know that

$$\boxed{v^2 = \mu (a^2 - x^2)}$$



Suppose that the particle moves from A (rest) to point O

we know that

$$\boxed{v = \frac{dx}{dt}}$$

$$v = \sqrt{\mu (a^2 - x^2)}$$

$$\therefore v = \frac{dx}{dt}$$

$$\frac{dx}{dt} = \sqrt{\mu (a^2 - x^2)}$$

$\because$ The particle from A to the direction of O then distance x decreases.

$$\frac{dx}{dt} = -\sqrt{\mu (a^2 - x^2)}$$

$$\frac{dx}{dt} = -\sqrt{\mu} \cdot \sqrt{a^2 - x^2}$$

$$-\frac{1}{\sqrt{\mu}} \cdot \frac{dx}{\sqrt{a^2 - x^2}} = dt$$

Integrating both sides

$$-\frac{1}{\sqrt{\mu}} \int \frac{dx}{\sqrt{a^2 - x^2}} = \int dt$$

Remaining part of last class

By equation of motion

$$\frac{d^2x}{dt^2} = -\mu x \quad (- \text{ means that } x \text{ is decreasing})$$

$$\frac{d^2x}{dt^2} = -\mu x \qquad \text{———} \quad ①$$

Equation ① is multiplied by $2 \cdot \frac{dx}{dt}$

$$2 \cdot \frac{dx}{dt} \cdot \frac{d^2x}{dt^2} = -2\mu x \cdot \frac{dx}{dt} \qquad \text{———} \quad ①$$

Let $\frac{dx}{dt} = P$ Then $\frac{d^2x}{dt^2} = \frac{dP}{dt}$

$$2\int \frac{dx}{dt}\left(\frac{d^2x}{dt}\right)dt = -2\mu \int x \cdot \frac{dx}{\cancel{dt}}\cancel{dt}$$

$$2\int P\frac{dP}{\cancel{dt}}\cancel{dt} = -2\mu \int x \cdot dx$$

$$\cancel{2}\frac{P^2}{\cancel{2}} = -\cancel{2}\mu \frac{x^2}{\cancel{2}} + c$$

$$P^2 = -\mu x^2 + c$$

$$\therefore P = \frac{dx}{dt}$$

$$\left(\frac{dx}{dt}\right)^2 = -\mu x^2 + c \qquad \boxed{\frac{dx}{dt} = v}$$

$$v^2 = -\mu x^2 + c$$

At point A

$v = 0$, $x = a$

$$0 = -\mu a^2 + c \qquad c = \mu a^2$$

$$v^2 = -\mu a^2 - \mu \cancel{x}^2$$

$$v^2 = \mu a^2 - \mu x^2$$

$$\boxed{v^2 = \mu(a^2 - x^2)} \qquad \text{———} \quad ⓐ$$

$$C - \frac{1}{\sqrt{\mu}}\left(-\cos^{-1}\frac{x}{a}\right) = t$$ ④

$$C + \frac{1}{\sqrt{\mu}}\cos^{-1}\frac{x}{a} = t$$

$$t = \frac{1}{\sqrt{\mu}}\cos^{-1}\frac{x}{a} + c$$

at point A

$x = a$. $t = 0$. $v = 0$

$$0 = \frac{1}{\sqrt{\mu}}\cos^{-1}\left(\frac{a}{a}\right) + c$$

$$0 = \frac{1}{\mu} \times 0 + c$$

$$c = 0$$

$$\boxed{t = \frac{1}{\sqrt{\mu}}\cos^{-1}\frac{x}{a}}$$ or, $\boxed{\sqrt{\mu}\cdot t = \cos^{-1}\frac{x}{a}}$

or, $\boxed{\cos t\cdot\sqrt{\mu} = \frac{x}{a}}$

let the particle takes time t_1 to reach at point O. Then $t = t_1$

$x = 0$

$$t_1 = \frac{1}{\sqrt{\mu}} \times \cos^{-1}(0)$$

$$t_1 = \frac{1}{\sqrt{\mu}} \times \cos^{-1}(0)$$

$$t_1 = \frac{1}{\sqrt{\mu}} \times \frac{\pi}{2}$$

$$\boxed{t_1 = \frac{\pi}{2\sqrt{\mu}}}$$

$$t = \frac{1}{\sqrt{\mu}}\cos^{-1}\left(\frac{x}{a}\right)$$

when the particle start from O and comes to rest at a point A. Then

(5)

$\frac{dx}{dt} = \sqrt{\mu(a^2 - x)}$ then distance increases.

$$\frac{dx}{dt} = \sqrt{\mu(a^2 - x^2)} \quad \text{or,} \quad \frac{dx}{dt} = \sqrt{\mu} \cdot \sqrt{a^2 - x^2}$$

$$\frac{dx}{\sqrt{\mu}\sqrt{a^2 - x^2}} = dt$$

$$\frac{1}{\sqrt{\mu}} \int \frac{dx}{\sqrt{a^2 - x^2}} = \int dt$$

$$\int dt = \frac{1}{\sqrt{\mu}} \int \frac{dx}{\sqrt{a^2 - x^2}}$$

$$t = \frac{1}{\sqrt{\mu}} \sin^{-1}\left(\frac{x}{a}\right) + c$$

$$t = \frac{1}{\sqrt{\mu}} \sin^{-1} \frac{x}{a} + c$$

On point 0 $x = 0$, $t = 0$

$$0 = \frac{1}{\sqrt{\mu}} \sin^{-1}(0) + c$$

$$\Rightarrow 0 = 0 + c$$

$$c = 0$$

$$t = \frac{1}{\sqrt{\mu}} \sin^{-1} \frac{x}{a}$$

$$\sqrt{\mu}\, t = \sin^{-1} \frac{x}{a}$$

$$\sin \sqrt{\mu} \cdot t = \frac{x}{a}$$

$$a \cdot \sin \sqrt{\mu}\, t = x$$

$$\boxed{x = a \cdot \sin\left(\sqrt{\mu} \cdot t\right)}$$