

## COSETS.

**Cosets:** A subset of a mathematical group that consists of all the products obtained by multiplying either on the right or the left a fixed element of the group by each of the elements of a given subgroup.

\* In mathematics, specifically group theory, a subgroup  $H$  of a group  $G$  may be used to decompose the underlying set of  $G$  into disjoint equal-size subsets called Cosets. There are left cosets and right cosets. Cosets

have the same number of elements as does  $H$ . Furthermore,  $H$  itself is both a left coset and a right coset. The number of left cosets of  $H$  in  $G$  is equal to the number of right cosets of  $H$  in  $G$ . This common value is called the index of  $H$  in  $G$  and is usually denoted by  $[G:H]$ .

Cosets are a basic tool in the study of groups; for example, they play a central role in Lagrange's theorem that states that for any finite

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group  $G$ , the number of elements of every subgroup  $H$  of  $G$  divides the number of elements of  $G$ . Cosets of a particular type of subgroup (a normal subgroup) can be used as the elements of another group called a quotient group or factor group. Cosets also appear in other areas of mathematics such as vector spaces and error-correcting codes.

### Definition:

Let  $H$  be a subgroup of the group  $G$  whose operation is written multiplicatively. Given an element  $g$  of  $G$ , the left cosets of  $H$  in  $G$  are the sets obtained by multiplying each element of  $H$  by a fixed element  $g$  of  $G$  (where  $g$  is the left factor). In symbols these are,

$$gH = \{ gh : h \text{ an element of } H \} \text{ for each } g \text{ in } G.$$

The right cosets are defined similarly, except that the element  $g$  is now a right factor, that is,