MPHYCC-2

Unit : 4.

# Subgroup :

**Subgroup:** A group whose elements are contained in another group is called a subgroup.

Thus a non-empty subset $H \in G$ is a sub-group of $G$, if $H$ satisfies all the four postulates of the group. So, if $H \in G$ be called a sub-group of $G$, we have:

(1) The product of any pair of elements of $H$ be in $H$. This means $a, b \in H \Rightarrow ab \in H$.

(2) $H$ contains the inverse of each of its elements, i.e. $a \in H \Rightarrow a^{-1} \in H$.

(3) Identity element $E \in H$.

The associative law holds for $G$ and thus holds for $H$, and this takes fourth postulate into account. These three conditions for $H$ to be a sub group of $G$ can be combined into a single condition:

$$a, b \in H \Rightarrow ab^{-1} \in H, \quad \text{---} (2)$$

**Proof:-**

We suppose that subset $H \in G$ is a sub-group of $G$ and hence, all these three conditions are satisfied.

Thus, $a, b \in H \Rightarrow ab \in H$.

then $a, b \in H$ means $b^{-1} \in H$: so $a \in H$ and $b^{-1} \in H$. Hence $ab^{-1} \in H$.

Now, suppose (eqn. 2) holds good since $H$ is not empty, there is an element $h \in H$.

By (eqn. 2) $hh^{-1} \in H$.

Hence, $E \in H$.

If $a \in H$, then $Ea^{-1} \in H$, i.e., $a^{-1} \in H$.

If $a, b \in H$, then $b^{-1} \in H$, and therefore

$$a(b^{-1})^{-1} = ab \in H.$$

Consequently, Condition (eqn. 2) includes all the three conditions and hence it is sufficient to be satisfied for a sub-set $H$ to be called a sub-group of a given group $G$.