Change in entropy in irreversible cycle:

The efficiency of a Carnot reversible cycle working between absolute temperatures T_1 and T_2 is given by

$$e = 1 - \frac{Q_2}{Q_1},$$

where Q_1 is the amount of heat taken in at temperature T_1 and Q_2 that given up at temperature T_2. Since the cycle is reversible, we have $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$. Thus

$$e = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}.$$

This is the max^m possible efficiency of an engine working between temp T_1 and T_2. If the cycle of the engine be irreversible, the efficiency will be lowered. Thus, in this case

$$e = 1 - \frac{Q_2}{Q_1} < 1 - \frac{T_2}{T_1}$$

or, $$\frac{Q_2}{Q_1} > \frac{T_2}{T_1}$$

or, $$\frac{Q_2}{T_2} > \frac{Q_1}{T_1}.$$

Now, during a complete irreversible cycle, the entropy of the source decreases by Q_1/T_1, while that of the sink increases by Q_2/T_2. The net chage in the entropy, of the working substance in this case also is zero because when the cycle is completed, the working substance recovers its initial state. Thus the total increase in the entropy of the system (working substance) plus the surroundings (source and sink) is

$$\Delta S = \frac{Q_2}{T_2} - \frac{Q_1}{T_1},$$

which is positive since $\frac{Q_2}{T_2} > \frac{Q_1}{T_1}$.

Thus, in an irreversible cycle the entropy of the system plus its surroundings always increases.

In general, we can write

$$\Delta S\,(\text{universe}) \geq 0,$$

Where the equality sign holds for reversible process and the inequality sign for irreversible process. Here the word universe means system + surroundings.