II
p-A

Describe with theory a method to determine the ratio of the thermal conductivity to the electrical conductivity of a metal.

In 1853 Widemann and Franz gave empirical law that at a particular temperature, the ratio of thermal and electrical conductivity is the same for metals. This law is known as Widemann-Franz law also. In 1872 Lorentz showed that the ratio (K/σ) is proportional to the absolute temperature T.

Drude tried to explain this law by assuming that the free electrons of metals are responsible for both the thermal and electrical conduction. He argued that the free electrons in a metal behave like gas molecules. They collide with the atoms of the metal and achieve an equilibrium state.

Let n be the number of free electrons per unit volume, λ be the mean free path, and $\bar{c}$ be the average speed.

Let a temperature gradient be set up in the z-direction. The energy of the free electrons will then vary from layer to layer and the free electrons will have the energy of the layer from which they came. A free electron will transport its energy through a distance λ. It can be shown that the total transfer of energy per unit area of a layer is

$$-\frac{1}{3} n \bar{c} \lambda \frac{dE}{dz} \quad \text{———} \quad (1)$$

If K be the thermal conductivity and dT/dz be the temperature gradient, then the flow of energy across unit area will be

$$-JK \frac{dT}{dz} \quad \text{———} \quad (2)$$

where J is the mechanical equivalent of heat.

so.

$$JK\frac{dT}{dz} = \frac{1}{3} n \bar{c} \lambda \frac{d\bar{E}}{dT} \cdot \frac{dT}{dz}$$

or, $$K = \frac{1}{3} \frac{n \bar{c} \lambda}{J} \frac{d\bar{E}}{dT} \qquad (3)$$

Since the free electrons possess only the translational energy

$$\bar{E} = \frac{3}{2} K_0 T$$

$$\therefore \frac{d\vec{E}}{dT} = \frac{3}{2} K_0 .$$

Substituting it in eqn. (3) we get.

$$K = \frac{1}{2} \frac{n \bar{c} \lambda}{J} K_0 \qquad (4)$$

Now let us consider the conduction of electricity in a metallic wire. If the electric intensity along the wire be X, then every free electron of the wire will experience an acceleration eX/m where e and m are respectively the charge and mass of the electron. The free electron gets accelerated during the successive collisions and parts with its acquired velocity on collisions with the atom. So the drift velocity at the beginning of the path is zero. In traversing a path of length λ, the electrons take on average time $\lambda/\bar{c}$. So the drift velocity at the end of the free path is

$$\frac{\lambda}{\bar{c}} \frac{Xe}{m} .$$

Hence the average drift velocity

$$u = \frac{1}{2} \frac{\lambda}{\bar{c}} \frac{Xe}{m} \qquad (5)$$

So the current density (charge flowing per unit area per unit time)

$$j = neu = ne\frac{1}{2} \frac{\lambda}{\bar{c}} \frac{Xe}{m} .$$

Hence the electrical conductivity

$$\sigma = \frac{j}{X} = \frac{ne^2\lambda}{2m\bar{c}} \qquad (6)$$

Now since the average energy

$$\frac{1}{2} m \bar{c}^2 = \frac{3}{2} K_0 T$$

So $$m \bar{c} = \frac{3 K_0 T}{\bar{c}}$$

Hence $$\sigma = \frac{n e^2 \lambda \bar{c}}{6 K_0 T} \quad \text{———} \quad (7)$$

From equations (4) & (7) we get.

$$\frac{K}{\sigma T} = \frac{3}{2}\left(\frac{K_0}{e}\right)^2 = \text{constant} \quad \text{———} \quad (8)$$

which gives the law of Wiedemann-Franz law and of Lorentz. This eqn: is satisfied at ordinary temperatures. But at low temperatures the value of $K/\sigma T$ falls off. At low temperatures both the thermal and electrical conductivities decrease as the temperature is decreased. So it follows that with decreasing temperatures the electrical conductivity increases much more rapidly than the thermal conductivity.

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