## Hermitian operator :

An operator F representing an observable quantity must, for every state $\psi$, then

$$\langle F \rangle = (\psi, F\psi) \quad \text{——— (1)}$$

which is a real number. Consequently, F must satisfy the condition.

$$(\psi, F\psi) = (\psi, F\psi)^* = (F\psi, \psi) \quad \text{——— (2)}$$

For every function $\psi$ to which it may be applied. A linear operator which obeys rule of eqn. (2) is called a Hermitian operator. The rule is sufficient to insure that the eigenvalues of F are real, for if $\psi$ is an eigenfunction of F belonging to the eigenvalue $\lambda$, then.

$$(\psi, F\psi) = (\psi, \lambda\psi) = \lambda(\psi, \psi);$$

according to Eqns. (2) this expression is also equal to

$$(F\psi, \psi) = (\lambda\psi, \psi) = \lambda^*(\psi, \psi).$$

and since $(\psi, \psi) \neq 0$

$$\lambda = \lambda^*$$

i.e, $\lambda$ is real.

generally, a Hermitian operator satisfies the relation.

$$(\psi, F\phi) = (F\psi, \phi) \quad \text{——— (3)}$$

That is, the operator can be applied to either factor in the scalar product ③

Equation ~~2~~ 11.

$$\frac{\hbar}{i}(\psi^* \psi)\Big|_{-\infty}^{\infty} = 0 \quad \text{---} \quad ④$$

proves that the operator $P_{op}$ is Hermitian.

and eqn.

$$\langle p^2 \rangle = \frac{\int p^2 |a(p)|^2 dp}{\int |a(p)|^2 dp} = \frac{\int |(\hbar/i)(d\psi/dx)|^2 dx}{\int |\psi|^2 dx} \quad \text{---} \quad ⑤$$

Eqn. ⑤ is a special case of eqn. ③, namely

$$(\psi, P^2\psi) = (P\psi, P\psi).$$

It is clear from these examples that the Hermitian property is associated in an essential way with the boundary conditions imposed upon the functions $\psi$ and is not to be regarded as a property of the operator symbol itself (independent of the class of functions to which it is applied.)

$\Rightarrow$ It will now be shown that two eigenfunctions of a Hermitian operator belonging to different eigenvalues, are orthogonal.

Let $A$ be a Hermitian operator, and $\psi_1$ and $\psi_2$ two eigenfunctions of $A$ such that

$$A\psi_1 = \alpha_1\psi_1 , \qquad A\psi_2 = \alpha_2\psi_2 .$$

Forming the scalar product of the first of these relations with $\psi_2$, we have,

$$(\psi_2 , A\psi_1) = \alpha_1 (\psi_2 , \psi_1) \quad \text{—— (6)}$$

But . $A$ is Hermitian, and eqn (3) yields.

$$(\psi_2 , A\psi_1) = (A\psi_2 , \psi_1) = \alpha_2^* (\psi_2 , \psi_1)$$

$$= \alpha_2 (\psi_2 , \psi_1) , \quad \text{—— (7)}$$

Where the last equality holds because $\alpha_2$ is real. comparision of eqns (6) & (7) shows that,

$$(\alpha_2 - \alpha_1)(\psi_2 , \psi_1) = 0 \quad \text{—— (8)}$$

and since it has been assumed that $\alpha_2 \neq \alpha_1$ ,

$$(\psi_2 , \psi_1) = 0$$

This theorem guarantees that the orthogonality relations $(\psi_m , \psi_n) = \delta_{mn}$ are satisfied for a set of normalized eigenfunctions belonging to different eigenvalues $\alpha_n$.