Dr. ... B.Sc. Phy(H) Paper VII - C

Explain the fine structure of spectral lines of hydrogen atom.

Niel Bohr was the first to give a satisfactory atomic model of hydrogen atom.

Hydrogen atom consists of positively charged nucleus of charge e, surrounded by an electron moving in a stationary orbit. In order to give slightly greater generality, let us write the nuclear charge Ze, even though Z = 1 for hydrogen atom.

We know, from first postulate of Bohr's theory;

$$r = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{mv^2} \quad \text{——— (1)}$$

From Bohr's second postulate; we have,

$$v = \frac{nh}{2\pi m r} \quad \text{——— (2)}$$

Substituting the value of v from eqn. (2) in eqn. (1); we get

$$r = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{m(nh/2\pi m r)^2} = \frac{1}{4\pi\epsilon_0}\frac{Ze^2 \times 4\pi^2 m^2 r^2}{m n^2 h^2}$$

∴ This gives

$$r = n^2 \frac{\epsilon_0 h^2}{\pi m Z e^2} \quad \text{——— (3)}$$

This eqn. gives the radius of nth stationary orbit; we denote it by r_n. Thus,

$$r_n = n^2 \frac{\epsilon_0 h^2}{\pi m Ze^2} \quad \text{——— (4)}$$

If v_n is the velocity of the electron in n^{th} orbit, the Kinetic energy of the electron in n^{th} orbit is

$$T_n = \frac{1}{2} m v_n^2 \quad \text{——— (5)}$$

For nth or orbit eqn may be written as

$$\frac{m v_n^2}{r_n} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r_n^2}$$

using this, we get the K.E of the electron in nth orbit

$$T_n = \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{2 r_n} \quad \text{——— (6)}$$

In addition to this K.E the electron has potential energy due to electrostatic attraction of the nucleus given by P.E in nth orbit

$$U_n = \frac{1}{4\pi\epsilon_0} \frac{Ze(-e)}{r_n} = -\frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r_n} \quad \text{— (7)}$$

∴ The total energy of the electron in nth orbit is given by:

$$E_n = T_n + U_n$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{2 r_n} - \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r_n}$$

$$= -\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{2 r_n} \quad \text{——— (8)}$$

Substituting value of r_n from eqn (4) in eqn (8) we get,

$$E_n = -\frac{mZ^2e^4}{8\epsilon_0^2 h^2} \cdot \frac{1}{n^2} \quad \text{———} \quad (9)$$

$$\therefore \quad E_n = -\frac{Z^2 R h c}{n^2} \quad \text{———} \quad (10)$$

Where, $R = \dfrac{me^4}{8\epsilon_0^2 c h^3}$ = Rydberg Constant.