MPHYCC-2
Unit: 4

GROUP THEORY

Group theory has become an important part of modern physical and chemical analysis. Its mathematical methods are useful in many physical problems. Its most obvious application consists in the classification of crystals and polyatomic molecules according to a group of the appropriate symmetry.

For the development of group theory and to define a group, we have to define a set as follows:

A set is a collection of objects which are called the elements of the set.

If x is an element of a set A, we write $x \in A$, which means 'x belongs to A'. If x is not an element of A, we write $x \notin A$ which means 'x does not belong to A'.

If P is a property, the set of all objects with the property P will be denoted by $\{x \mid x \text{ satisfies } P\}$. The set which does not contain any element is called the empty set and is denoted by ϕ.

For example a set

$$S = \left\{ x \;\middle|\; \begin{array}{l} x \text{ is real} \\ \text{and } x^2 = -1 \end{array} \right\} = \phi \text{ is an empty set.}$$

Let X and Y be two sets. If every element of X is an element of Y, we say X is a subset of Y, and write $X \leq Y$. It is clear that if $X \leq Y$ and $Y \leq X$, we must have $X = Y$.

If X and Y are two sets, then by the union of X and Y denoted by $X \cup Y$, we mean a set

$$\{z \mid z \in X \text{ or } z \in Y\}.$$

The intersection of X and Y, denoted by $X \cap Y$ is defined as the set

$$\{z \mid z \in X \text{ and } z \in Y\}.$$ If $X \cap Y = \phi$ (empty set), we say X and Y are disjoint.

If $X \leq Y$, we define the complement of X in Y, denoted by $Y - X$, as a set

$$\{z \mid z \in Y \text{ and } z \notin X\}.$$