P-9. QM MECHANICS Kumar

Ref 164 Schiff

NOTES ON

Dirac's BRA and KET Notation:-

We describe a state function which is denoted by ψ_α, a_α, or b_α, by a Ket or Ket vector $|\alpha\rangle$, and the hermition adjoint state $\psi_\alpha^\dagger$, $a_\alpha^\dagger$, or $b_\alpha^\dagger$ by a bra or bra vector $\langle\alpha|$. This notation is called Dirac BRA and KET Notation. The inner product of two state vectors is taken by;

$$\psi_\alpha^\dagger \psi_\beta = \langle\alpha|\beta\rangle \quad \text{——} \quad (1)$$

and is called bracket expression, which is a number. The first and last three letters of bracket provides the name for the two kinds of state vectors.

Operation on a ket vector from the left with Ω produces another ket vector.

$$\Omega|\beta\rangle = |\beta'\rangle \quad \text{——} \quad (2)$$

and operation on a bra vector from the right with Ω produces another bra vector

$$\langle\alpha|\Omega = \langle\alpha''|$$

The matrix element of Ω between states α and β is a number and can be written in any of the equal forms. Of which the first and last are the commonly used notation.

$$\Omega_{\alpha\beta} = \int \psi_\alpha^*(r)\,\Omega\,\psi_\beta(r)\,d^3r = \int [\Omega^\dagger \psi_\alpha(r)]^* \psi_\beta(r)\,d^3r$$

$$= (\psi_\alpha, \Omega\psi_\beta)$$

$$= (\Omega^\dagger \psi_\alpha, \psi_\beta)$$

$$= \langle \alpha | \beta' \rangle.$$

$$= \langle \alpha'' | \beta \rangle$$

$$= \langle \alpha | \Omega | \beta \rangle \quad \text{—— (3)}$$

The matrix element of the hermition adjoint operator $\Omega^\dagger$ are given by.

$$(\Omega^\dagger)_{\beta\alpha} = \Omega^*_{\alpha\beta} = \langle \beta | \Omega^\dagger | \alpha \rangle$$

$$= \langle \alpha | \Omega | \beta \rangle^*$$

From eqn (2) & (3) ; Ω is the unit operator:

$$\langle \alpha | 1 | \beta \rangle = \langle \alpha | \beta \rangle$$

For bracket expression of the unitary matrix W ;

$$W_{k\mu} = \int u_k^*(r)\, v_\mu(r)\, d^3r$$ by identifying ψ_α with u_k and ψ_β with v_μ ;

$$\boxed{W_{k\mu} = \langle k | \mu \rangle = \langle \mu | k \rangle^*} \quad \text{—— (4)}$$