⇒ De-Morgan's theorem:-

Theorem I:- The complement of the sum of two or more variables is equal to the product of the complements of the variables.

If A & B are two variables.

$$\overline{A+B} = \overline{A}\cdot\overline{B} \quad \text{——— (1)}$$

Theorem II:- The complement of the product of two or more variables is equal to the sum of the complements of the variables.

$$\overline{A\cdot B} = \overline{A} + \overline{B} \quad \text{——— (2)}$$

→ To prove $\overline{A+B} = \overline{A}\cdot\overline{B}$

(i) When $A = 0$, $B = 0$.

$\overline{A+B} = \overline{0+0} = \overline{0} = 1$

and $\overline{A}\cdot\overline{B} = \overline{0}\cdot\overline{0} = 1\cdot1 = 1$

Hence. $\overline{A+B} = \overline{A}\cdot\overline{B}$.

(ii) When $A=0$, $B=1$, $\overline{A+B} = \overline{0+1} = \overline{1} = 0$

and $\overline{A}\cdot\overline{B} = \overline{0}\cdot\overline{1} = 1\cdot0 = 0$

Hence. $\overline{A+B} = \overline{A}\cdot\overline{B}$.

(iii) When $A=1$, $B=0$, $\overline{A+B} = \overline{1+0} = \overline{1} = 0$.

and $\overline{A}\cdot\overline{B} = \overline{1}\cdot\overline{0} = 0\cdot1 = 0$.

Hence, $\overline{A+B} = \overline{A}\cdot\overline{B}$

(iv) When $A=1$, $B=1$, $\overline{A+B} = \overline{1+1} = \overline{1} = 0$.

and $\overline{A}\cdot\overline{B} = \overline{1}\cdot\overline{1} = 0\cdot0 = 0$ Hence. $\overline{A+B} = \overline{A}\cdot\overline{B}$.

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→ To prove $\overline{A \cdot B} = \overline{A} + \overline{B}$

(i) When $A = 0$, $B = 0$. $\overline{A \cdot B} = \overline{0 \cdot 0} = \overline{0} = 1$

and $\overline{A} + \overline{B} = \overline{0} + \overline{0} = 1 + 1 = 1$

Hence. $\overline{A \cdot B} = \overline{A} + \overline{B}$.

(ii) When, $A = 0$. $B = 1$.

$\overline{A \cdot B} = \overline{0 \cdot 1} = \overline{0} = 1$

and $\overline{A} + \overline{B} = \overline{0} + \overline{1} = 1 + 0 = 1$

Hence. $\overline{A \cdot B} = \overline{A} + \overline{B}$.

(iii) When. $A = 1$, $B = 0$. $\overline{A \cdot B} = \overline{1 \cdot 0} = \overline{0} = 1$

and $\overline{A} + \overline{B} = \overline{1} + \overline{0} = 0 + 1 = 1$

Hence. $\overline{A \cdot B} = \overline{A} + \overline{B}$

(iv) When. $A = 1$ & $B = 1$

$\overline{A \cdot B} = \overline{1 \cdot 1} = \overline{1} = 0$

and $\overline{A} + \overline{B} = \overline{1} + \overline{1} = 0 + 0 = 0$.

Hence, $\overline{A \cdot B} = \overline{A} + \overline{B}$.

Thus De-morgon's theorem proved.

Additional laws and theorem of Boolean Algebra

Algebra :- Like ordinary algebra, Commutative, Associative & distributive laws are varified for Boolean Algebra.

Commutative laws :- $A + B = B + A$

$A \cdot B = B \cdot A$

Associative laws :- $A + (B + C) = (A + B) + C$

$A(BC) = (AB)C$

These two groups of law can be thro
ght of in terms of OR and AND gates.
Each of these identities can be proved
by substituting the two possible va
of A i,e 0 and 1; on each side of th
identity. In each case, the left-hand
will equal to the right-hand side. Aga
the double-complement operation give

$$\overline{\overline{A}} = A$$

→ Some other useful theorem:-

(i) $A + AB = A$

(ii) $A + \overline{A}B = A + B$

(iii) $A(A+B) = A$

(iv) $A(\overline{A}+B) = AB$

(v) $(A+B)(A+C) = A+BC$

(vi) $(A+B)(\overline{A}+C) = AC+\overline{A}B$.

putting the two values of 0 & 1; ab
identities can be proved:-