Exp. Solve the initial value problem

$$y' \cot x + y = 2 \; ; \; y(\pi/3) = 0$$

Solution If $y \neq 2$ and $\cot x \neq 0$ the differential Equ. can be written as

$$\frac{dy}{y-2} + \tan x \, dx = 0$$

Integrating

$$\int \frac{dy}{y-2} + \int \tan x \, dx = c$$

$$\log(y-2) + (-\log \cos x) = \log c$$

$$\log(y-2) = \log c \cos x$$

$$y - 2 = c_1 \cos x$$

$$y = 2 + c_1 \cos x \quad \text{———} \quad (1)$$

Where $c_1 = \log c$ is an arbitrary constant.

initial condition $x = \pi/3$ & $y = 0$ putting in Eqn (1)

$$0 = 2 + c_1 \cos \pi/3$$

$$c_1 \cos \pi/3 = -2$$

$$c_1 \cdot 1/2 = -2 \Rightarrow c_1 = -4$$

Hence the desired particular solution is given explicity by $y = 2 - 4\cos x$

Exp. Solve the Equation

$$(x^3+1)\,dy = y\,dx$$

$$(x^3+1)\,dy - y\,dx = 0$$

If $x \neq -1$ and $y \neq 0$ we can write this equation in the form

$$\frac{dy}{y} = \frac{dx}{x^3+1}$$

Solution- Since $x^3+1 = (x+1)(x^2-x+1)$

we have using partial fractions

$$\frac{1}{x^3+1} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$$

where A, B, C are undetermined constants.

$$1 = A(x^2-x+1) + (Bx+C)(x+1)$$

$$1 = x^2(A+B) + x(-A+B+C) + (A+C)$$

$$A+B = 0$$

$$-A+B+C = 0$$

$$A+C = 0$$

Solving these we find $A = 1/3$, $B = -1/3$, $C = 2/3$

on substituting these values and integrating we have

$$\int \frac{dy}{y} = \frac{1}{3}\int \frac{dx}{x+1} - \frac{1}{3}\int \frac{x-2}{x^2-x+1}\,dx$$

$$\int \frac{dy}{y} = \frac{1}{3}\int \frac{dx}{x+1} - \frac{1}{3}\int \frac{(x-1/2)\,d(x-1/2)}{(x-1/2)^2+3/4} + \frac{1}{2}\int \frac{dx}{(x-1/2)^2}$$

$$\log y = \frac{1}{3}\log(x+1) - \frac{1}{6}\log(x^2-x+1) + \frac{1}{\sqrt{3}}\tan^{-1}\frac{2x-}{\sqrt{3}}$$

$$\log y = \frac{1}{6} \ln \frac{(x+1)^2}{x^2-x+1} + \frac{1}{\sqrt{3}} \tan^{-1} \frac{2x-1}{\sqrt{3}} + c$$

where c is an arbitrary constant. The original has also the singular solutions $x=-1$ $(y \neq 0)$ and $y=0$ $(x \neq -1)$

If a differential equation can be written in the form

$$y' = f(ax+by)$$

then we put $z = ax+by$ and have

$$\frac{dz}{dx} = a + bf(z)$$

$$\frac{dz}{a+bf(z)} - dx = 0$$

in which the variables are separable.

→ Solve these Questions.

Que 1- $y' = \dfrac{x^2}{y(1+x^3)}$

Que 2- $\dfrac{dy}{dx} = \dfrac{3x^2+4x+2}{2(y-1)}$; $y(0) = -1$

Que 3- $y' + \sin^2(x+y) = 0$