* Determination of nth derivative of Algebraic rational functions:- To find out the nth derivative of any algebraic rational function.

→ Firstly, we find out the partial fractions of that function.

→ Partial fractions may be of two forms:-

(i) In which the denominator contains linear factors only.

(ii) other in which the denominator contains the linear as well as quadratic factors.

→ If the denominator in the partial fraction consists of linear (first degree) expression, then its nth derivative can be found out by direct formula.

→ But if the denominator in the partial fraction consists of an expression of second degree, then its nth derivative is found out by the application of De-Moivre's Theorem.

$\Rightarrow$ $[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

Exp. Find y_n, when $y = \frac{1}{6x^2+11x+3}$

Solution Find out the partial fraction of

$$\frac{1}{6x^2+11x+3} = \frac{1}{6x^2+9x+2x+3}$$

$$= \frac{1}{(3x+1)(2x+3)}$$

$$\frac{1}{(3x+1)(2x+3)} = \frac{A}{(3x+1)} + \frac{B}{(2x+3)}$$

$$= \frac{A(2x+3)+B(3x+1)}{(3x+1)(2x+3)}$$

$\therefore$ $A(2x+3)+B(3x+1) = 1$

Putting $x = -1/3$, $A(-\frac{2}{3}+3) + B(-\frac{3}{3}+1) = 1$

$$\frac{7A}{3} = 1 \Rightarrow A = \frac{3}{7}$$

Again putting $x = -3/2$, $A(2\times-\frac{3}{2}+3) + B(3\times-\frac{3}{2}+1) = 1$

$$A(0) + B(-\frac{7}{2}) = 1$$

$$B = -2/7$$

$$\frac{1}{6x^2+11x+3} = \frac{3}{7}\cdot\frac{1}{(3x+1)} - \frac{2}{7}\cdot\frac{1}{(2x+3)}$$

Hence $y = \frac{3}{7}\frac{1}{(3x+1)} - \frac{2}{7}\cdot\frac{1}{(2x+3)}$

From the formula. if $y = \frac{1}{ax+b}$

$$\Rightarrow y_n = \frac{a^n (-1)^n \cdot \lfloor n}{(ax+b)^{n+1}}$$

$$\therefore y_n = \frac{3}{7} \cdot \frac{(3)^n \cdot (-1)^n \cdot \lfloor n}{(3x+1)^{n+1}} - \frac{2}{7} \cdot \frac{2^n \cdot (-1)^n \cdot \lfloor n}{(2x+3)^{n+1}}$$

$$= \frac{(-1)^n \cdot \lfloor n}{7} \left[\frac{3 \cdot 3^n}{(3x+1)^{n+1}} - \frac{2 \cdot 2^n}{(2x+3)^{n+1}} \right]$$

$$y_n = \frac{(-1)^n \cdot \lfloor n}{7} \left[\frac{3^{n+1}}{(3x+1)^{n+1}} - \frac{2^{n+1}}{(2x+3)^{n+1}} \right]$$

$=$

<u>Exp.</u> Find y_n, when $y = \frac{x^n}{1+x}$

<u>Sol.</u> let $1+x = t$ so that $x = t-1$

$$y = \frac{(t-1)^n}{t} = \frac{t^n - {}^nc_1 t^{n-1} + {}^nc_2 t^{n-2} - \cdots + (-1)^n}{t}$$

$$y = t^{n-1} - {}^nc_1 t^{n-2} + {}^nc_2 t^{n-3} - \cdots + \frac{(-1)^n}{t}$$

$\because$ Expression of $(a+b)^n = a^n + {}^nc_1 a^{n-1} b + {}^nc_2 a^{n-2} b^2 + \cdots$

$\left({}^nc_r = \frac{\lfloor n}{\lfloor r \lfloor n-r}\right)$

Now

$$y = (x+1)^{n-1} - \frac{n}{\lfloor 1} \cdot (x+1)^{n-2} + \frac{n(n-1)}{\lfloor 2} (x+1)^{n-3} - \cdots + \frac{(-1)^n}{x+1}$$

$$\therefore y_n = 0 + 0 + 0 - 0 + \cdots (-1)^n \cdot \frac{(-1)^n \cdot \lfloor n}{(x+1)^{n+1}}$$

$$y_n = \frac{\lfloor n}{(x+1)^{n+1}}$$