Matrices

1. Definitions:

(i) Matrix: Any ordered rectangular array of $m \cdot n$ numbers (or scalars belonging to a field F) arranged in m rows and n columns is called $m \times n$ matrix (over F). Thus

$$A = \begin{matrix} R_1 \to \\ R_2 \to \\ \\ \\ R_m \to \end{matrix} \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix}$$

Ex:- $\begin{bmatrix} 2 & 3 \\ a_{11} & a_{12} \\ 4 & 5 \\ a_{21} & a_{22} \\ 6 & 7 \\ a_{31} & a_{32} \end{bmatrix}$

A 2d array of elements

No. of Rows = 3
No. of Columns = 2

Horizontal → Rows, vertical → Columns, etc are

is an $m \times n$ matrix (where a_{11}, a_{12} etc are numbers). It is also denoted by $[a_{ij}]_{m,n}$ or $(a_{ij})_{m,n}$ or simply by (a_{ij}). The suffix i takes up values $1, 2, \ldots m$ and the suffix j takes up values $1, 2, \ldots n$. Also a_{ij} is called the $(i,j)^{th}$ element of the matrix A.

No. of elements in a matrix = No. of Rows × No. of Columns

(ii) Square matrix: A matrix having equal number of rows and columns is called a square matrix of order n where n is the number of rows.

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(iii) Row matrix: A matrix consisting only of one row, i.e. an $1 \times n$ matrix is called a row matrix (or row vector)

(iv) Column matrix: A matrix consisting of only one column i.e. $m \times 1$ matrix is called a column matrix.

(v) **Zero matrix** : A matrix all of whose elements are 0 is called a zero matrix. A zero matrix may be rectangular or square.

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(vi) **Diagonal matrix** : A square matrix all of whose non-principal diagonal elements are 0 is called a diagonal matrix. For example :

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ are diagonal matrix.

(vii) **Scalar matrix** : If all the diagonal elements in a diagonal matrix are equal, the matrix is called a scalar matrix. for example

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ are scalar matrices

(viii) **Unit matrix** : A square matrix whose principal diagonal elements are each 1 and all all other elements are 0 is called a unit matrix.

For example $[1]$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ etc. are unit matrices of order 1, 2, 3, resp. These are denoted by $I_1, I_2, I_3, \ldots\ldots$