

Linear programming

Definitions

1. Point sets :- Point sets are sets whose elements are points or vectors in E^n (n -dimensional Euclidean space)

For example

(i) a linear equation in two variables x_1, x_2 i.e. $a_1 x_1 + a_2 x_2 = b$ represents a line in two dimensions. This line may be considered as a set of those points (x_1, x_2) which satisfy $a_1 x_1 + a_2 x_2 = b$. This set of points can be written as

$$S_1 = \{(x_1, x_2) : a_1 x_1 + a_2 x_2 = b\}$$

(ii) Consider the set of points lying inside a circle of unit radius with centre at the origin, in two dimensional space (E^2). Obviously the points (x_1, x_2) of this set satisfy the inequality $x_1^2 + x_2^2 < 1$. This set of points can be written as

$$S_2 = \{(x_1, x_2) : x_1^2 + x_2^2 < 1\}$$

2. Hypersphere. A hypersphere A in E^n with centre at a and radius $\epsilon > 0$ is defined to be the set of points

$$X = \{x : |x - a| = \epsilon\}$$

i.e. the equation of a hypersphere in E^n is

$$(x_1 - a_1)^2 + (x_2 - a_2)^2 + \dots + (x_n - a_n)^2 = \epsilon^2$$

where $a = (a_1, a_2, \dots, a_n)$, $x = (x_1, x_2, \dots, x_n)$

which represent a circle in E^2 and sphere in E^3 .

3. An ϵ neighbourhood :- An ϵ -neighbourhood about the point a is defined as the set of points lying inside the hypersphere with centre at a and radius $\epsilon > 0$.
i.e. the ϵ neighbourhood about the point is the set of points
$$X = \{x : |x - a| < \epsilon\}$$

4. An Interior point :- A point a is an interior point of the set S if there exists an ϵ -neighbourhood about a which contains only points of the set S .
An interior point of S must be an element of S .

5. A Boundary point :- A point a is a boundary point of the set S if every ϵ neighbourhood about a ($\epsilon > 0$ however small) contains points which are in the set and the points which are not in the set.

A boundary point of S must be an element of S .

6. An open set :- A set S is said to be an open set if it contains only interior point.