Normed Vector space :- Let V(F) be Vector space. For any u∈V define

norm of u = $\|u\| = \sqrt{\langle u,u \rangle}$

Then the vector space with this definition of norm is called normed vector space if the following conditions are satisfied.

(i) $\|u\| \geq 0$, $\|u\| = 0$ iff $u = 0$

(ii) $\|\alpha u\| = |\alpha| \, \|u\|$, $\forall \alpha \in F$ & $u \in V$

(iii) $\|u+v\| \leq \|u\| + \|v\|$, $u, v \in V$

Def. of Invariant - Let T: V→V be linear operator and W a subspaces of a vector space V(F). Then W is said to be invariant under T if T maps W into itself i.e if any w∈W

$\Rightarrow T(w) \in W$

In the presesent case T induces a linear operat $\hat{T}: W \to W$ defined by

$$\hat{T}(w) = T(w) \quad \forall w \in W.$$

Unitary operator - A linear operator on an inner product space V(F) is said to be unitary operator if (i) T is one-one onto

(ii) $\langle T(u), T(v) \rangle = \langle u, v \rangle$, $\forall u, v \in V$

A unitary operator on an Euclidean space V(R) is called orthogonal operator.

* T is unitary iff $TT^* = T^*T = I$