D’Alembert’s Principle – Lect. 03

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Conservation of Energy

- We return to the conservation of energy from the point of view of D'Alembert's principle. Let's start by considering the virtual work associated with a collection of particles in Cartesian coordinates

$$(\vec{\mathbf{F}}^a - \sum_i m_i \ddot{\vec{\mathbf{r}}}_i) \cdot \delta \vec{\mathbf{r}} \qquad (8)$$

- Next assume that the force can be written as the gradient of a scalar, the virtual work becomes

$$(\delta V + \sum_i m_i \ddot{\vec{\mathbf{r}}}_i) \cdot \delta \vec{\mathbf{r}} = 0 \qquad (9)$$

Conservation of Energy (contd.)

- The virtual displacement can be any arbitrary displacement that is consistent with the constraints.
- We will select it to be a real infinitesimal displacement

$$(dV + \sum_i m_i \ddot{\vec{\mathbf{r}}}_i) \cdot d\vec{\mathbf{r}} = 0 \qquad (10)$$

- The second term can be converted to a perfect differential of a scalar

$$\sum_i m_i \ddot{\vec{\mathbf{r}}}_i \cdot d\vec{\mathbf{r}} = \sum_i m_i \ddot{\vec{\mathbf{r}}}_i \cdot d\dot{\vec{\mathbf{r}}}\, dt = \sum_i \frac{d}{dt}\left(\frac{1}{2} m_i \dot{\vec{\mathbf{r}}}_i \cdot \dot{\vec{\mathbf{r}}}_i\right) dt = \sum_i \frac{dT_i}{dt} dt = dT \qquad (11)$$

Conservation of Energy (contd.)

- where T is the kinetic energy as previously defined. Based on this expression, the virtual work becomes

$$dV + dT = d(T + V) = 0 \Rightarrow T + V = E \qquad (12)$$

- Therefore, the sum of the kinetic and potential energy is a constant.
- The question we must ask ourselves before using this result is, under what conditions does this hold? The first condition is that the force be derivable from a scalar potential.
- The second condition requires the virtual displacements be the same as the real displacement.
- This condition is satisfied if the the problem is time independent. That is the constraints and the potential are scleronomic (time independent).

References

1. The Variational Principles of Mechanics, *C. Lanczos* pgs. xxi, xxix
2. Classical Dynamics, *D.T. Greenwood* chap. 1
3. Classical mechanics, J.C Upadhya chap . 2
4. Classical mechanics, S.K Sinha chap 1

Short Questions

- **Q. What is meant by principle of virtual work?**
- **Ans.** The principle of virtual work states that when an object is in equilibrium the virtual work done by the forces on the object will be equal to zero. The same is stated in Newton's laws which states that the applied forces are equal and opposite when at equilibrium.
- **Q. What is the statement of D'Alembert's principle?**
- **Ans.** It is stated as system of mass of particles, the sum of difference of the force acting on the system and the time derivatives of the momenta is zero when projected onto any virtual displacement.

Short Questions (contd.)

- **Q.What is meant by inertial force?**
- **Ans.** Inertial force is defined as the force which is in the opposite direction of the accelerating force and is equal to the product of the accelerating force and the mass of the body.

- **Q. What is the use of D’Alembert’s principle?**
- **Ans.** D’Alembert’s principle is used for analyzing the dynamic problem which can reduce it into astatic equilibrium problem.

Short Questions (contd.)

- **Q. What is virtual displacement?**
- **Ans.** A virtual displacement is defined as the instantaneous change in the coordinates of the system.

- **Q. What are the quantities related in Newton's second law of motion?**
- **Ans.** In Newton's second law there are two vector quantities, and they are force and acceleration.