D'Alembert's Principle – Lect. 02

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Application of D'Alembert's principle

- D'Alembert's principle is based on the principle of virtual work along with inertial forces.
- The following are the applications of D'Alembert's principle:
  - Mass falling under gravity
  - Parallel axis theorem
  - Frictionless vertical hoop with a bead

Case Example

- As an example let’s consider a wedge of mass M on a frictionless surface, with a block of mass m on the wedge (see Fig. 1). We will calculate the equations of motion for this system.

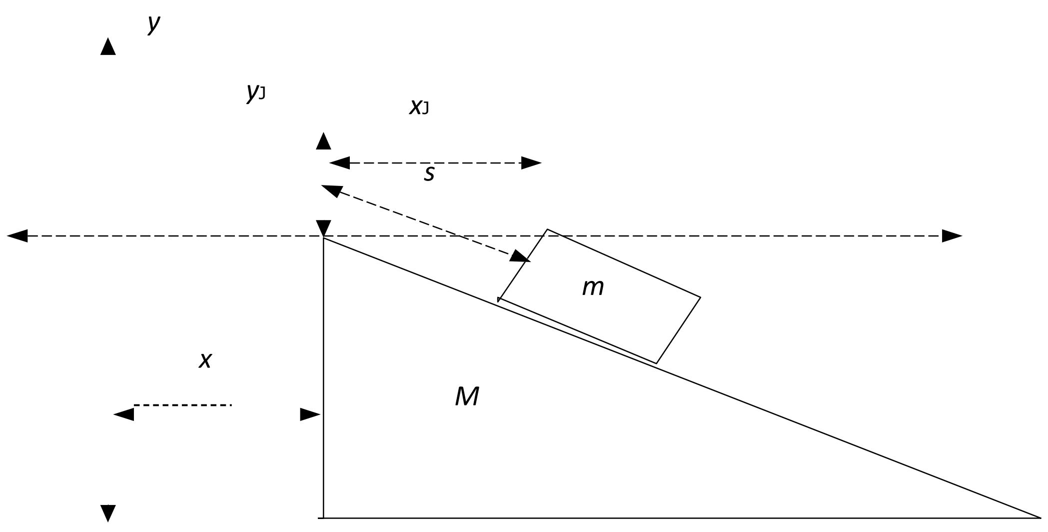

Fig. 1: Block sliding down a frictionless incline, with include also free to slide frictionlessly on a flat surface.

Case Example (contd.)

- Using the coordinate system specified in Fig 1, the virtual work consistent with the constraints, in Cartesian coordinates is

$$\delta W = -mg\delta y' - m(\ddot{x} + \ddot{x}')(\delta x + \delta x') - M\ddot{x}\delta x - m\ddot{y}'\delta y' = 0 \tag{4}$$

- Since the variables x' and y' are not independent of each other, because of the constraint of moving along the surface of the wedge, we apply the following transformation to take into account of the constraints

$$\left.\begin{aligned} x' &= s\cos\theta \\ y' &= -s\sin\theta \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} \delta x' &= \delta s\cos\theta \\ \delta y' &= -\delta s\sin\theta \\ \ddot{x}' &= \ddot{s}\cos\theta \\ \ddot{y}' &= -\ddot{s}\sin\theta \end{aligned}\right. \tag{5}$$

Case Example (contd.)

- Applying this transformation and grouping like terms together, the virtual work becomes

$$[mg \sin \theta - m(\ddot{s} + \ddot{x} \cos \theta)]\, \delta s - [M \ddot{x} + m(\ddot{s} \cos \theta + \ddot{x})]\, \delta x = 0 \qquad (6)$$

- Notice at this point, we reduced the virtual work such that there are only two independent variations, which is the number of degrees of freedom:
- The wedge is constrained to move in one dimension, as is the block on the wedge.
- Since the variations are independent of each other and arbitrary, the terms in brackets must independently be equal to zero.

Case Example (contd.)

- Therefore, the equations of motion are

$$mg \sin\theta - m(\ddot{s} + \ddot{x}\cos\theta) = 0 \qquad (7)$$

$$M\ddot{x} + m(\ddot{s}\cos\theta + \ddot{x}) = 0$$