D’Alembert’s Principle – Lect. 01

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Introduction

- D’Alembert’s principle is used for analyzing the dynamic problem which can reduce it into astatic equilibrium problem.
- For a system of mass of particles, the sum of difference of the force acting on the system and the time derivatives of the momenta is zero when projected onto any virtual displacement.
- It is also known as the Lagrange-D’Alembert principle, named after the French mathematician and physicist Jean le Rond D’Alembert.
- It is an alternative form of Newton’ssecond law of motion. According to the 2nd law of motion, F = ma, while it is represented as F – ma= 0 in D’Alembert’s law.
- So it can be said that the object is in equilibrium when a real force is acting on it. Here, F is the real force while -ma is the fictitious force known as inertial force.

Mathematical Representation of D'Alembert's Principle

- D'Alembert's principle can be explained mathematically in the following way:
- D'Alembert's principle introduces the force of inertia $\vec{I} = -m\vec{a}$, hereby converting problems of dynamics to problems of statics

$$\vec{F} = m\vec{a} \quad \Rightarrow \quad \vec{F} - m\vec{a} = 0 \quad \Rightarrow \quad \vec{F} + \vec{I} = 0 \qquad (1)$$

- where we show the transition from Newton to D'Alembert in this expression. The force $\vec{F}$ is sometimes referred to as the real force, which I will do so in these lectures to distinguish it from the inertial force.

Mathematical Representation of D’Alembert’s Principle (contd.)

- The requirement that the sum of all the forces at each particle be equal to zero is the necessary condition for static equilibrium.
- Since the principle of virtual work applies to systems in static equilibrium, we will apply it to this system of forces including the inertial force.
- The total work done by the forces in this system through an arbitrary virtual displacement in Cartesian coordinates is

$$\delta W = \sum_i \left[\vec{\mathbf{F}}_i^a + \vec{\mathbf{F}}_i^c - m_i \ddot{\vec{\mathbf{r}}}_i\right] \cdot \delta \vec{\mathbf{r}}_i = 0 \qquad (2)$$

Mathematical Representation of D'Alembert's Principle (contd.)

- where we have split the real forces into the applied and constraint forces.
- If the constraint forces are workless, and the virtual displacements reversible and consistent with the constraints, the total virtual work becomes

$$\delta W = \sum_i \left[\vec{\mathbf{F}}_i^a - m_i\ddot{\vec{\mathbf{r}}}_i\right] \cdot \delta\vec{\mathbf{r}}_i = 0 \qquad (3)$$

- This equation expands upon the principle of virtual work from static to dynamical system.
- Note, this equation applies to both rheonomic and scleronomic system, provided that the virtual displacements confirm to the instantaneous constraint.

Example

- 1D motion of rigid body: T – W = ma or T = W + ma where T is tension force of wire, W is weight of sample model and *ma* is acceleration, force.
- The 2D motion of rigid body: For an object moving in an x-y plane the following is the mathematical representation: F_i= -m $\ddot{r}_c$ where F_i is the total force applied on the ith place, m mass of the body and r_c is the position vector of the center of mass of the body.
- This is called as D'Alembert's principle.