Linear programming

Definitions :-

7. **A closed set :-** A set S is said to be a closed set if it contains all its boundary points.

8. **Lines :-** In E^n, the line through the two points x_1 and x_2, $x_1 \neq x_2$ is defined to be the set of points

$$X = \{x : x = \lambda x_2 + (1-\lambda) x_1, \text{ for all real } \lambda\}$$

9. **Line segments :-** In E^n, the line segment joining two points x_1 and x_2 is defined to be the set of points.

$$X = \{x : x = \lambda x_2 + (1-\lambda) x_1, 0 \leq \lambda \leq 1\}$$

10. **Hyperplane :-** A hyperplane is defined as the set of points satisfying

$$c_1 x_1 + c_2 x_2 + \cdots + c_n x_n = z \quad (\text{not all } c_i = 0)$$

$$cx = z$$

for prescribed values of $c_1, c_2, \cdots, c_n$ and

2. The vector c is called a normal to the hyperplane and $\pm \frac{c}{|c|}$ are called unit normals.

A hyperplane divides the whole space E^n into three mutually disjoint sets given by

$$X_1 = \{x : cx > z\}$$
$$X_2 = \{x : cx = z\}$$
$$X_3 = \{x : cx < z\}$$

The sets X_1 and X_2 are called open half spaces.

Definitions :-

1. Convex Combination :-

A Convex Combination of a finite number of points $x_1, x_2, \ldots, x_n$ is defined as a point.

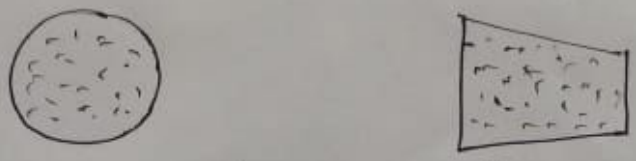
$$x = \lambda_1 x_1 + \lambda_2 x_2 + \cdots + \lambda_n x_n$$

where λ_i is real and ≥ 0, $\forall i$

and $\sum_{i=1}^{n} \lambda_i = 1$.

2. Convex set :-

A set of points is said to be Convex if for any two points in the set, x is also in the set. In other words a set is Convex if the Convex Combination of any two points in the set, is also in the set.



Convex sets

Non-Convex sets

3. Extreme point of a Convex set

A point X in a Convex set C is called an extreme point if X cannot be expressed as a Convex Combination of any two distinct points x_1 and x_2 in C.

Obviously, an extreme point is a boundary point of the set.

It is important to note that all boundary points of a convex set are not necessarily extreme points

4. Convex Hull :-

The convex hull C(x) of any given set of points X is the set of all convex combination of sets of points from X.

Ex:- If X is the set of eight vertices of a cube, then the convex hull Cf(x) is the whole cube.

5. Convex function :-

A function f(x) is said to be strictly convex at X if for any two other distinct points x_1 and x_2.

$$f\{\lambda x_1 + (1-\lambda) x_2\} < \lambda f(x_1) + (1-\lambda) f(x_2).$$

where on the other hand, a function f(x) is strictly concave if $-f(x)$ is strictly convex.

6. Convex Polyhedron :-

The set of all convex combinations of finite number of points is called the convex polyhedron generated by these points.

Ex:- The set of the area of a triangle is a convex polyhedron of its vertices.