Note :- since the convex combination of two points are infinite in number so from the above theorem we conclude that if a given L.P.P. has two feasible solution, then it has infinite number of feasible solutions.

Theorem 5 :- Every basic feasible solutions of the system $Ax = b$ $x \geq 0$ is an extreme point of the convex set of feasible solutions and conversely.

Proof :- To prove that every B.F.S. is an extreme point of the convex set of feasible solutions.

Let x be a B.F.S. of $Ax = b$ which is a n-component vector containing both zero (non-basic) and non-zero (basic) variables.

Let x_B and B be the vector of m basic variables and the matrix of vectors associated to basic variables in the B.F.S. x respectively then

$$x = \{x_B, 0\} \quad \text{——} \quad (1)$$

where 0 is a null vector of $(n-m)$ components.

and $$Ax = b \Rightarrow B \cdot x_B = b \quad \text{——} \quad (2)$$

Now we have to prove that x is an extreme points.

we shall prove this by using contradiction.

If x is not an extreme point then there exist two distinct points

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Theorem 4:- The set of all feasible solutions of a L.P.P. is a Convex set.

Proof:- Let X be the set of all feasible solutions of a L.P.P.

$$Ax = b,\ x \geq 0 \quad \text{———} \quad (1)$$

Case I If the set X has only one element, then X is Convex set. Hence the theorem is true in this Case.

Case II If the set X has at least two elements

Let x_1 and x_2 be any two distinct elements in X.

$$\therefore \quad Ax_1 = b, \qquad x_1 \geq 0$$

$$\text{and} \quad Ax_2 = b, \qquad x_2 \geq 0$$

If $x_3 = \lambda x_1 + (1-\lambda)x_2, \quad 0 \leq \lambda \leq 1$

then $Ax_3 = A\lambda x_1 + (1-\lambda)Ax_2$

$$= \lambda b + (1-\lambda)b = b$$

Also since $x_1 \geq 0$, $x_2 \geq 0$, $\lambda \geq 0$, $1-\lambda \geq 0$, as $\leq \lambda \leq 1$.

$$\therefore x_2 = \lambda x_1 + (1-\lambda)x_2 \geq 0$$

i.e. x_2 satisfy (1). Thus $x_2 = \lambda x_1 + (1-\lambda)x_2$ is also a F.S. and so belongs to set X.

But x_2 is a Convex Combination of any two distinct points x_1 and x_2 in X.

Hence by definition the set X is a Convex set.