Paper 1, TDC Part-1
Chapter– 4, Circuit Theorems
Lecture - 6

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Circuit Theorem

In this lecture we work out some problems based on Norton's Theorem

Example 1) Find the current through load resistor R_L using Norton's theorem for the below shown circuit.

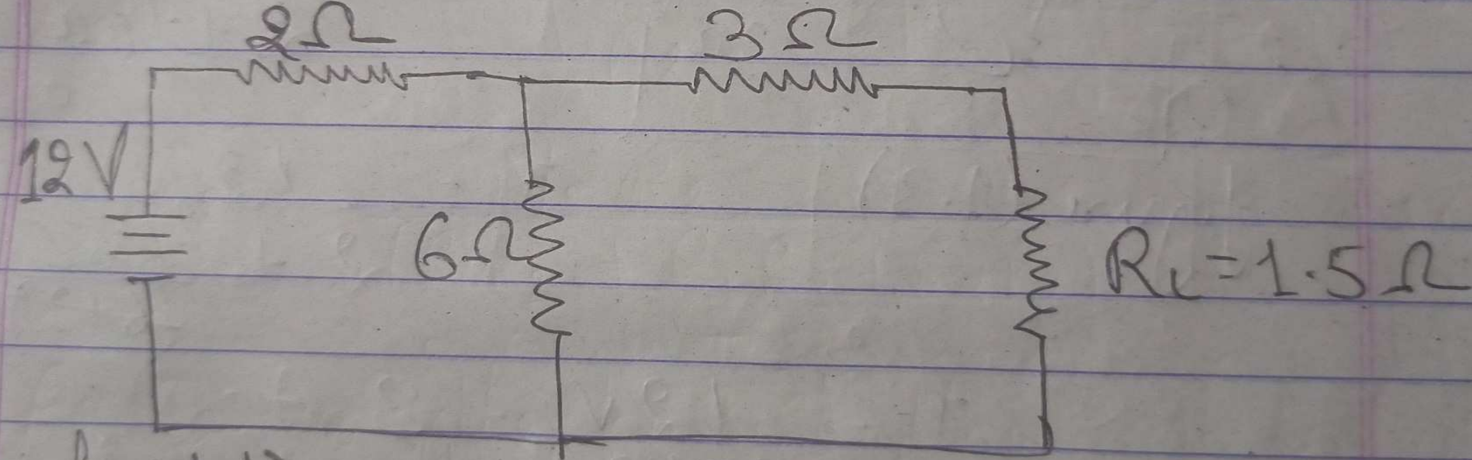


fig 1.1)

Soln: To apply, ~~NOR~~ Norton's theorem we

Circuit Theorem

Soln: To apply ~~NOR~~ Norton's theorem we will have to find short circuit current ($I_{sc} = I_N$) by removing load resistance R_L by short ckt. Redrawing the ckt.

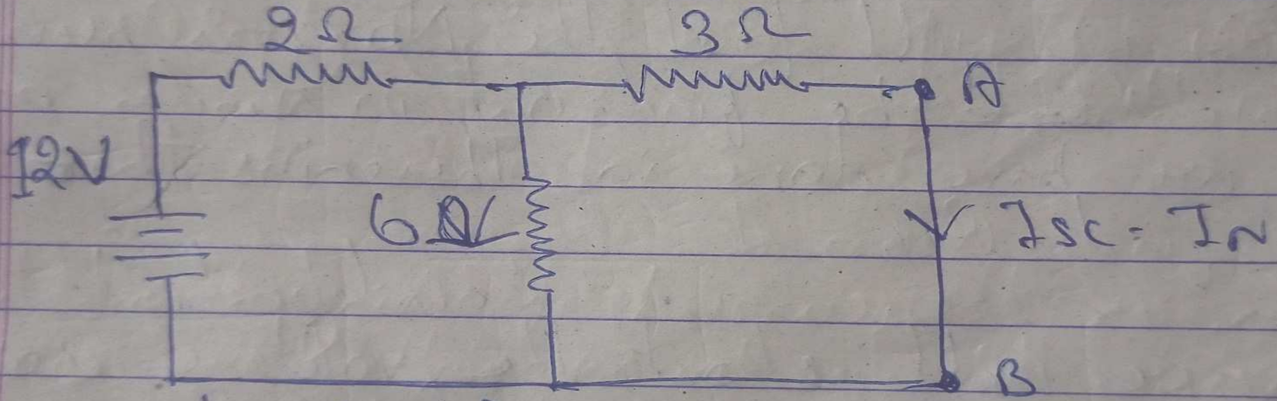


fig (1.2)

When we look into the ckt of figure 1.2 we see that

6Ω is ||rel with 3Ω resistor.

Circuit Theorem

then 2Ω will be in series with $(6\Omega \,||\, 3\Omega)$, So, finally we have

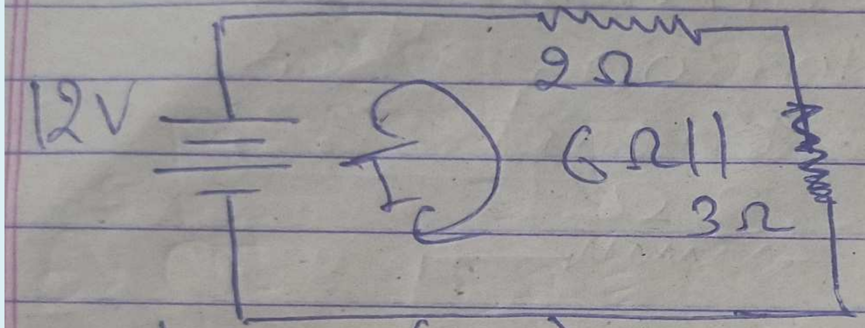


figure (1.3)

$$I = \frac{12V}{2 + (6\Omega || 3\Omega)}$$

$$I = \frac{12V}{2 + \frac{{}^{2}\cancel{6} \times \cancel{3}}{\cancel{(6+3)}\,\cancel{3}}}$$

$$I = \frac{12V}{4\Omega} = 3A$$

Now I_{sc} is current through 3Ω resistor So,

Circuit Theorem

$$I_{sc} = I_{3\Omega} = I_N = \frac{\cancel{3} \times \cancel{6}\ 2}{\cancel{(6+3)}\ 3} = 2A$$

Now we will find the Norton's resistance which ~~will~~ can be find out in similar manner as we have done in Thevenins theorem to find R_{th}.

So we will short the independent voltage source then.

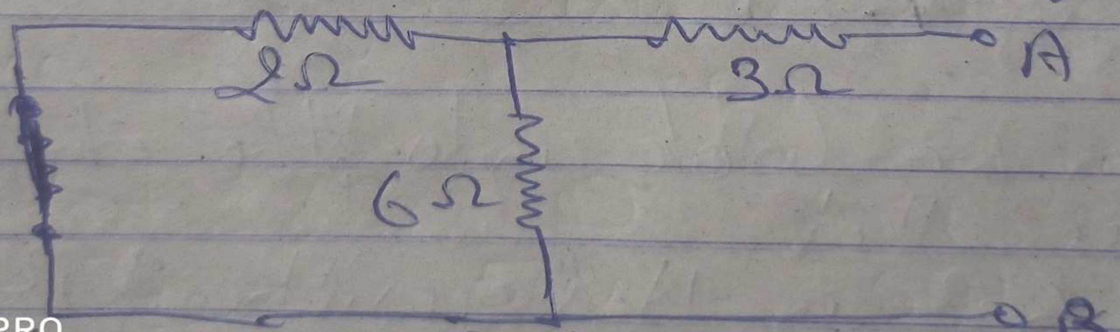


fig (1.4) To find R_N

Circuit Theorem

~~Here 2Ω~~

Here 2Ω & 6Ω are parallel and the resultand will be in series with 3Ω resistor

So,

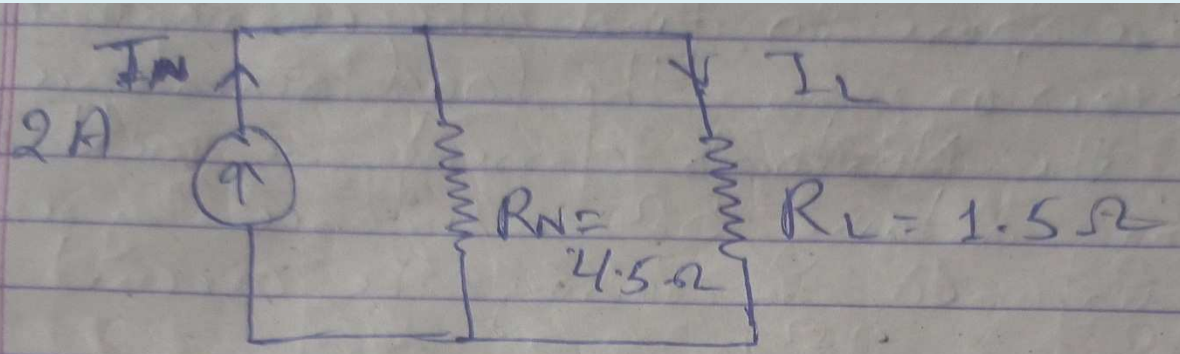
$$R_N = 3 + \frac{2 \times 6}{(2+6)} = 3 + \frac{\cancel{12}\,3}{\cancel{8}\,2}$$

$$= 9/2 = 4.5\,\Omega$$

~~So~~

So Norton's equivalent ckt to find current in load resistance R_L is.

Circuit Theorem



Current through load resistance (R_L) is I_L

$$I_L = \frac{2A \times 4.5\,\Omega}{(1.5+4.5)\,\Omega} = \frac{\cancel{2} \times \overset{1.5}{\cancel{4.5}}}{\cancel{6}\ \cancel{3}}\,A$$

$$\boxed{I_L = 1.5\,A}$$

Voltage across $R_L = 1.5\,A \times 1.5\,\Omega$

$= 2.25\,V$

Circuit Theorem

Example 2) Find the current through 5 Ω resistor for the ckt shown in below figure.

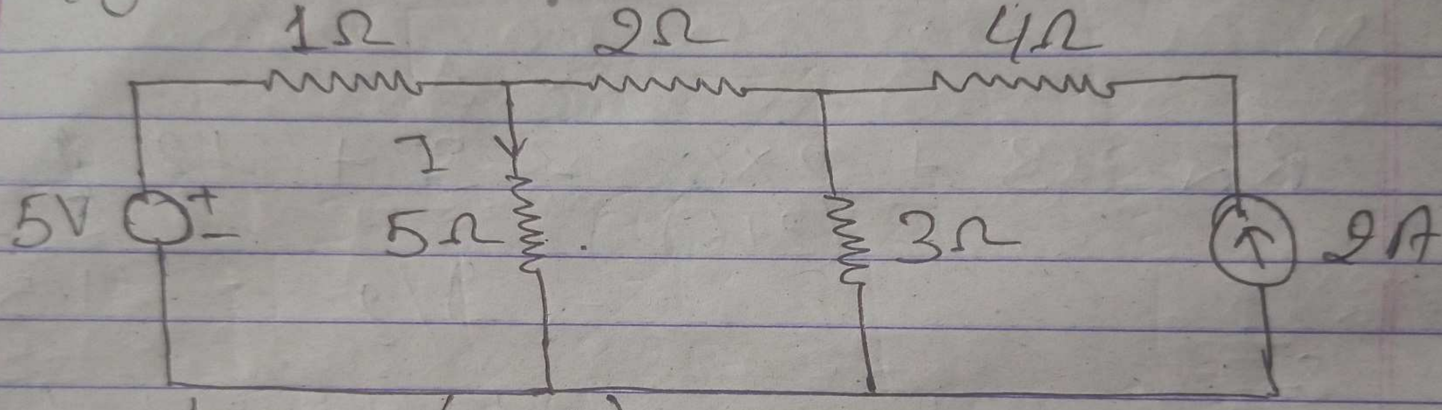
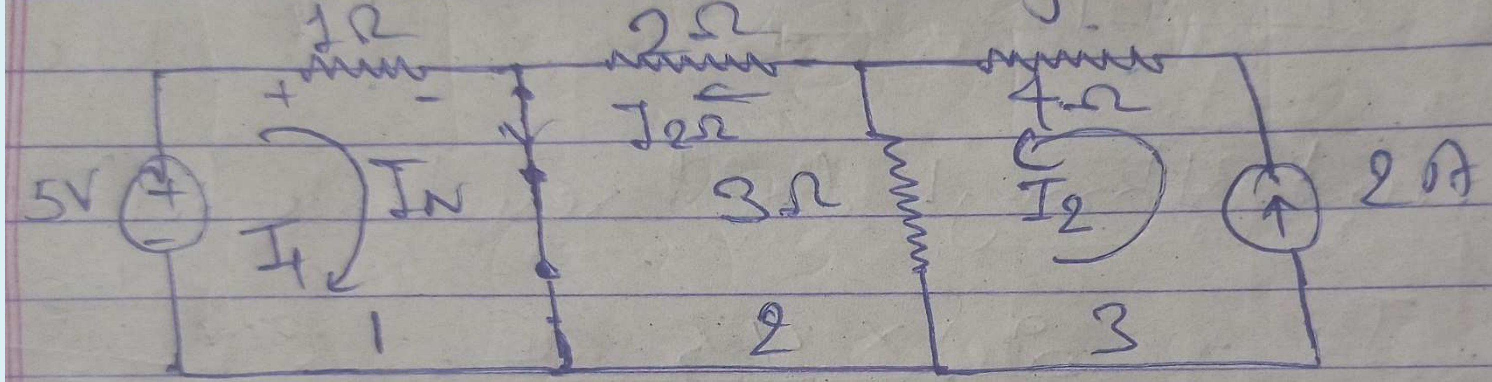


figure (2.1)

Soln. Since we have to find the current through 5Ω resistor so the load resistance $R_L = 5\Omega$

To find the Norton's current we will short the arm containg 5Ω resistance by removing 5Ω resistance. Drawing the circuit again.

5Ω resistance. Drawing the circuit again.



I_N is the sum of current flowing through the resistor 1Ω due to 5V voltage source and current through resistor 2Ω due to 2A source. So,

$$-5V + 1 \times I_1 = 0$$

$$I_{1\Omega} = \frac{5\,V}{1\,\Omega} = 5\,A$$

Circuit Theorem

$I_2 = 2A$

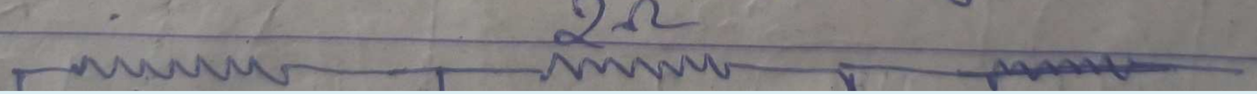
Current through 2Ω resistor due to 2A current ~~is given~~ source is

$$I_{2\Omega} = \frac{2 \times 3}{(3+2)} = \frac{6}{5} A$$

$$I_N = I_{1\Omega} + I_{2\Omega}$$

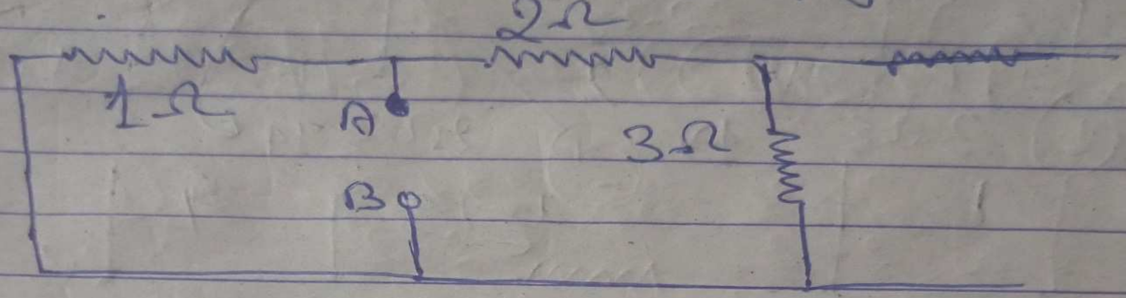
$$= \left(5 + \frac{6}{5}\right) A = \frac{31}{5} A = 6.2 A.$$

Now to find Norton's equivalent resistance we again draw the ckt of figure 2.1 as below



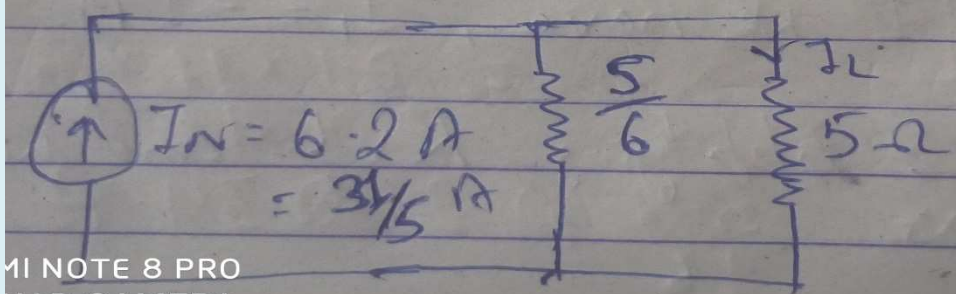
Circuit Theorem

... ckt of figure 2.1 as below



$$R_N = 1\Omega \parallel (2+3)\Omega$$
$$= 1\Omega \parallel 5\Omega$$
$$= \frac{5\times 1}{(5+1)} = \frac{5}{6}\ \Omega$$

Drawing the Norton's equivalent ckt with load resistance



$$I_L = \frac{31}{5} \times \frac{5}{6\times\left(\frac{5}{6}+5\right)}$$
$$= \frac{31}{6} \times \frac{6}{35}\ A = 0.89A$$

Circuit Theorem

Example 3) Find the current through 1 Ω resistor connected across AB, as shown in below diagram.

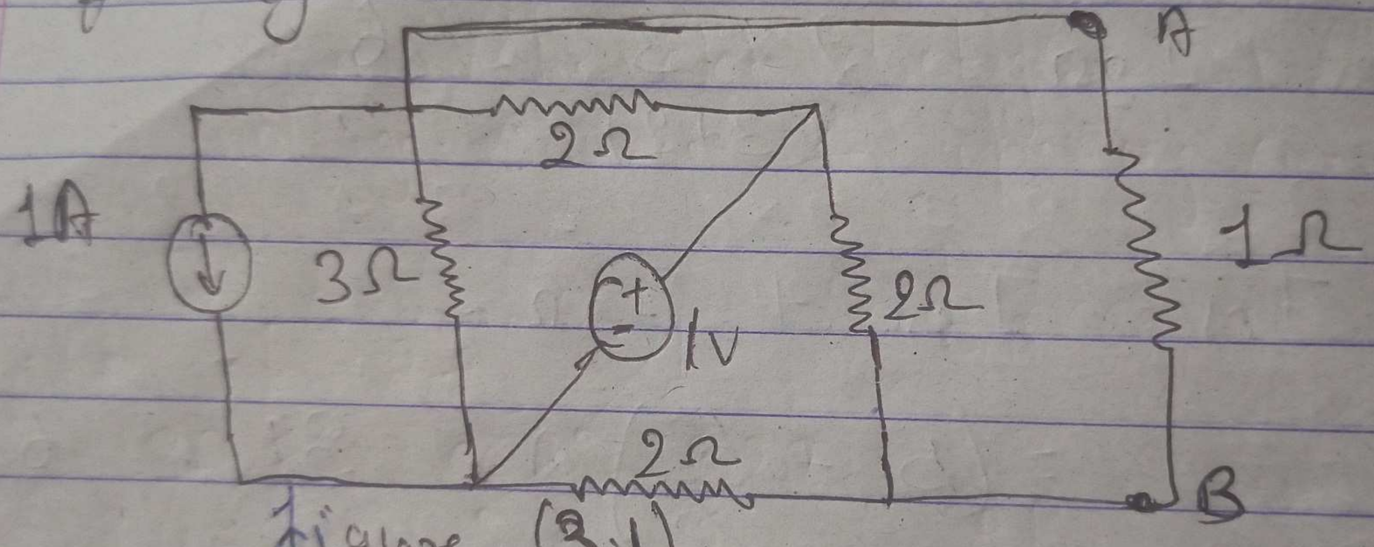


figure (3.1)

Solution. We draw the ckt of figure (3.1) as below to find short circuit current in branch AB

Circuit Theorem

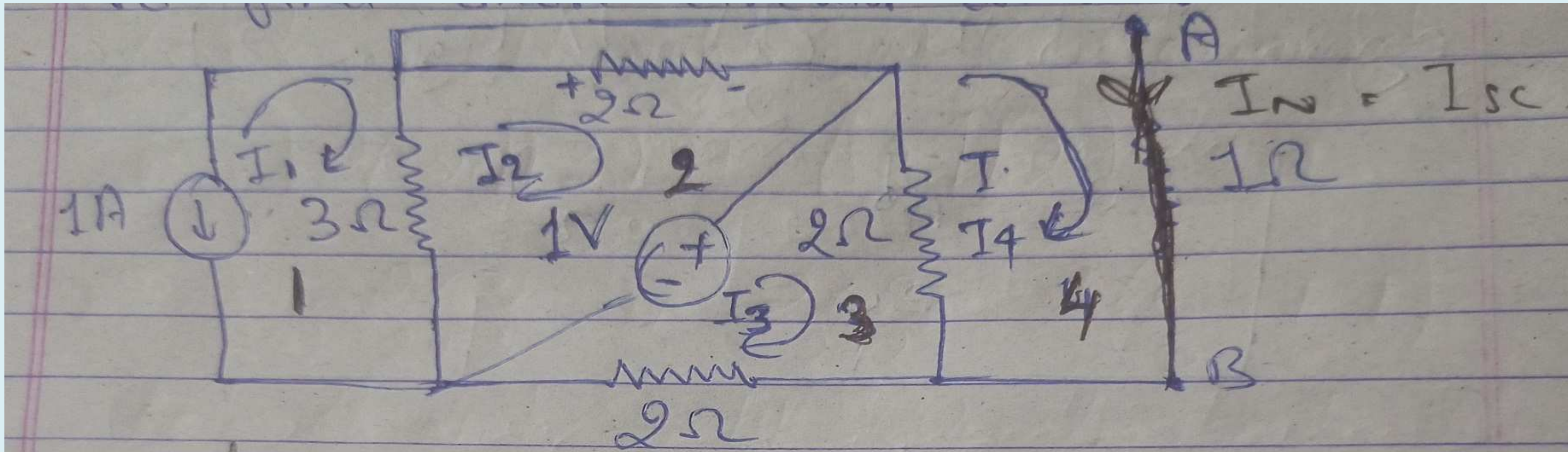


figure (3-2)

We consider, let current I_1, I_2, I_3 & I_4 flows through the loop 1, 2, 3 & 4 respectively as shown in figure (3-2).

from loop 1 we have, $I_1 = -1A$.

In loop 2, applying KVL and write them as.

Circuit Theorem

$$2I_2 + 3I_2 - 3I_1 - 2I_4 + 1 = 0$$

$$5I_2 - 3\times -1 - 2I_4 = -1$$

$$5I_2 + 3 - 2I_4 = -1$$

$$5I_2 - 2I_4 = -1 - 3 = -4$$

$$I_2 = \frac{-4 + 2I_4}{5}$$

Now applying KVL in loop 3, we have

$$2I_3 + 2I_3 - 2I_4 - 1 = 0$$

Circuit Theorem

$$4I_3 - 2I_4 = 1$$

$$I_3 = (1 + 2I_4)/4$$

Now Applying KVL in loop 4, we have

$$2I_4 + 2I_4 - 2I_2 - 2I_3 = 0$$

$$4I_4 - 2\left(\frac{-4 + 2I_4}{5}\right) - \frac{2(1 + 2I_4)}{4} = 0$$

$$\Rightarrow \frac{80I_4 + 32 - 16I_4 - 10 - 20I_4}{20} = 0$$

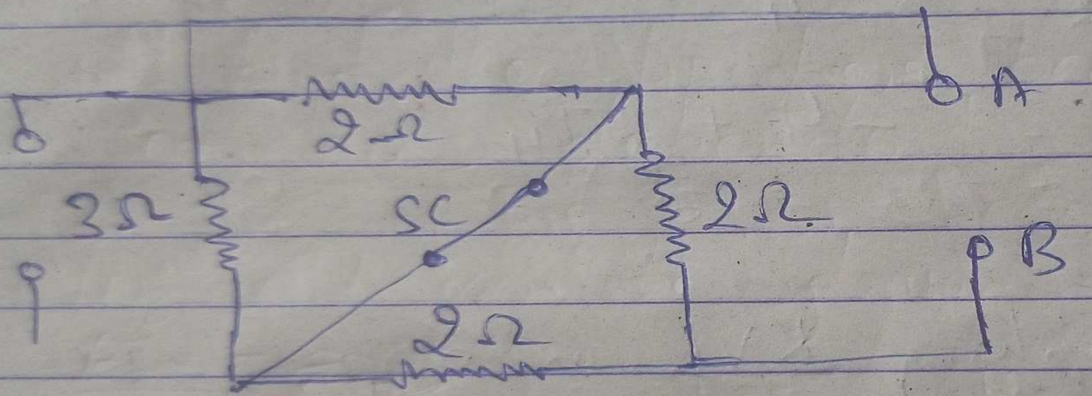
$$\Rightarrow 44I_4 + 22 = 0$$

or $I_4 = \frac{-22}{44}\,A = -0.5\,A$

Circuit Theorem

$I_4 = I_{sc} = I_N = -0.5\,A$

Now to find ~~current~~ Norton's resistor we again draw the ckt of figure 3.1, by short voltage source & opening current source



Looking through A B terminal,

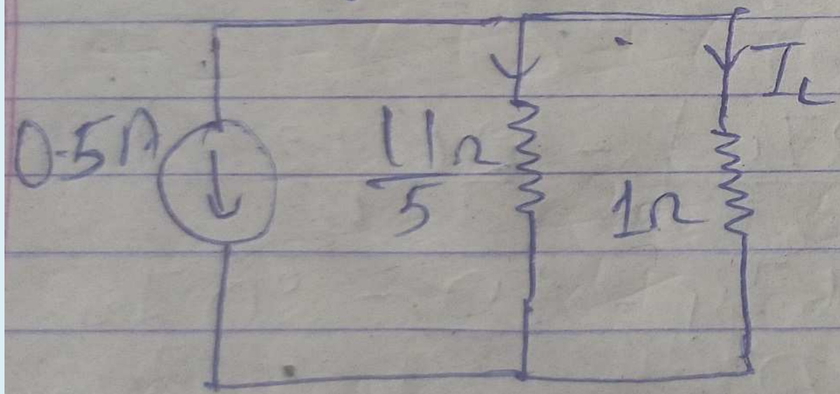
$R_N = (2 \| 3) + (2 \| 2)\ \Omega$

Circuit Theorem

$$= \frac{2\times3}{2+3} + \frac{2\times2}{2+2} = \left(\frac{6}{5}+1\right)\Omega$$

$$R_N = \frac{11}{5}\,\Omega$$

Drawing Norton's equivalent ckt.



$$I_L = \frac{\overset{0.1}{\cancel{0.5}} \times \frac{11}{\cancel{5}}}{\left(\frac{11}{5}+1\right)}$$

$$= \frac{0.1\times 11 \times 5}{16}$$

$$I_L = \frac{5.5}{16}\,A = 0.34\,A$$

Circuit Theorem

Example 4)

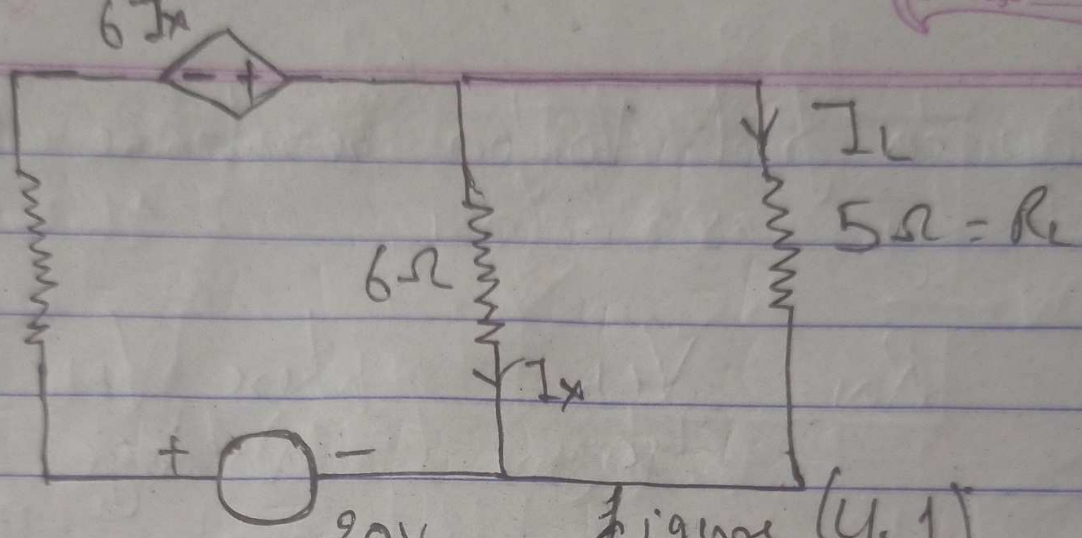


figure (4.1)

Find the current I_L through load resistance R_L using Norton's Theorem.

Solution) To find Nortons Current (I_N) we will short arm containing load resistance (5Ω) and draw the ckt.

Circuit Theorem

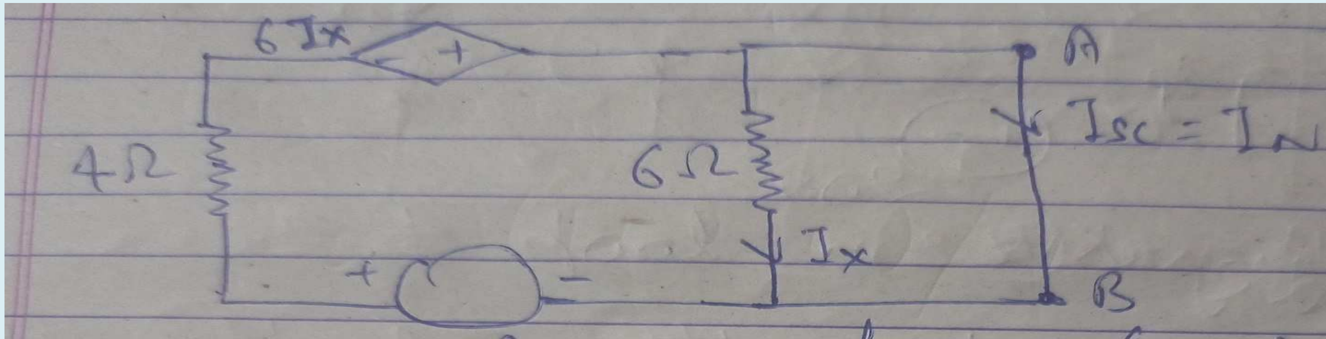
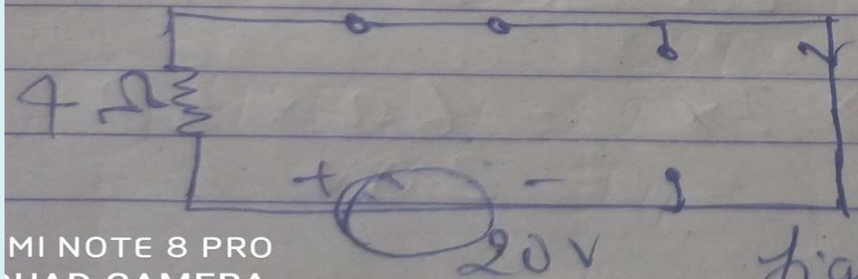


figure (4.2)

Since arm containing R_L is short now so no current will flow in 6 Ω resistor i.e. $I_x = 0$ and as $I_x = 0$ so, dependent voltage ~~vol~~ source $6I_x$ is also 0. Now ckt of figure (4.2) can be drawn as below



$$I_{sc} = I_N = \frac{20}{4} \text{ A}$$

$$\Rightarrow = 5 \text{ A}$$

figure (4.3)

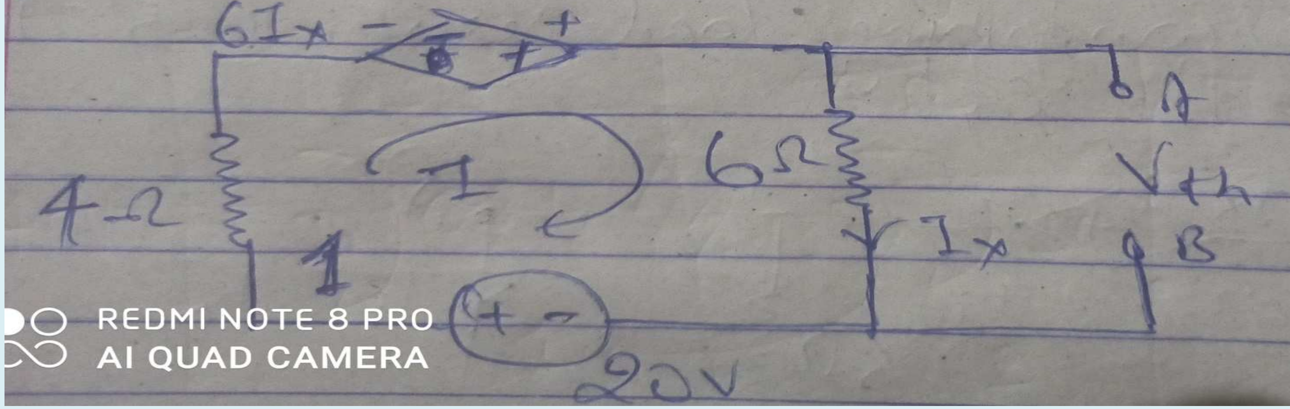
Circuit Theorem

Now to find Norton resistance we will find it by, as below,

$$R_N = \frac{V_{th}}{I_{sc}}$$ where V_{th} is Thevnins voltage.

We will find the V_{th} here because the ckt contains dependent voltage source.

for. V_{th} ckt can be drawn as.



Circuit Theorem

Applying KVL in loop 1.

Let current I flow in loop 1 then.

$I = I_x$

KVL equation for loop 1 is,

$$4I - \cancel{6I_x} + \cancel{6I_x} - 20 = 0$$

$$I = \frac{\cancel{20}}{\cancel{4}}\ 5\,A = I_x$$

$$V_{th} = 6\Omega \times I_x = 6\Omega \times 5A$$

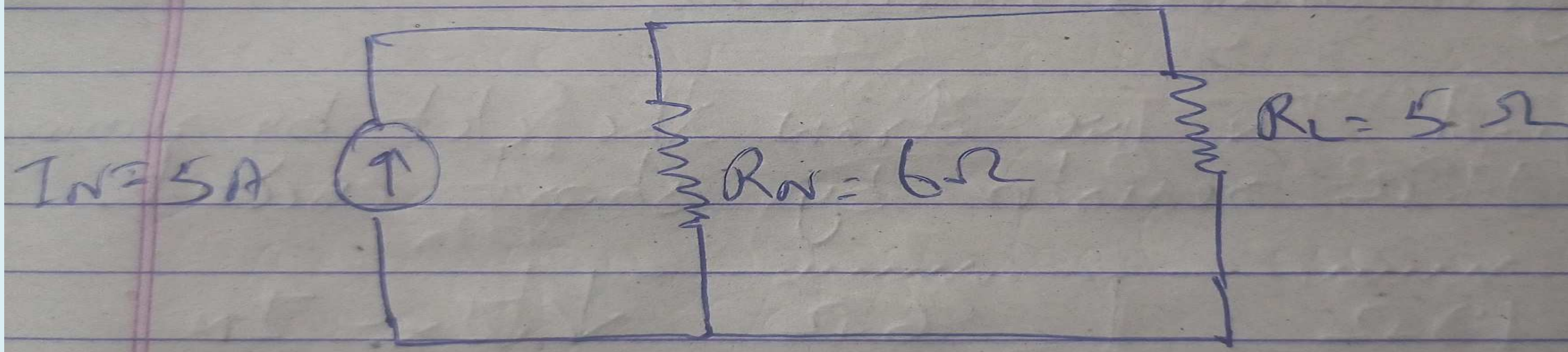
$$= 30V$$

Circuit Theorem

~~Norton's~~ Equ Norton's resistance.

$$R_N = \frac{V_{th}}{I_{sc}} = \frac{30\,V}{5\,A} = 6\,\Omega$$

Norton's equivalent ckt. is,

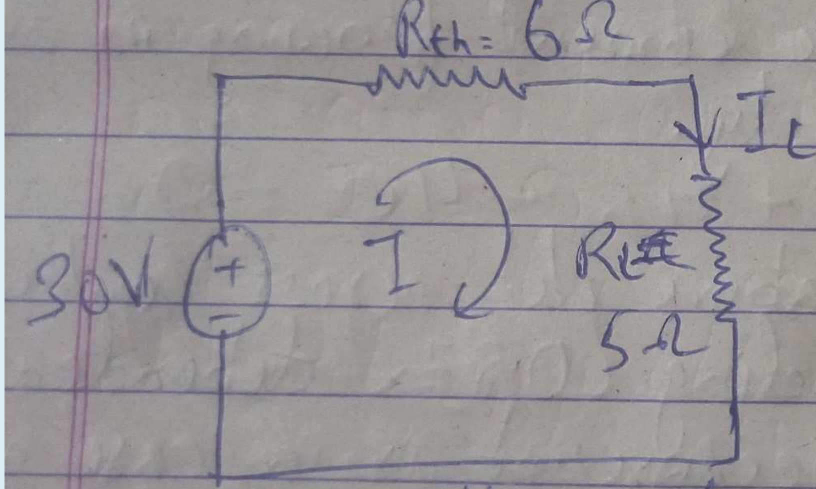


$$I_L = \frac{I_N \times R_N}{(R_N + R_L)} = \frac{5 \times 6}{(6+5)}$$

Circuit Theorem

$$= \frac{30}{11} A = 2.73$$

We can also draw Thevenin's Equivalent ckt as below.



$$I_L = I = \frac{30}{(6+5)} = \frac{30}{11}$$

$$= 2.73 A$$ as same as above.

So no matter which theorem has been applied the result will be same.

Circuit Theorem

Example 5)

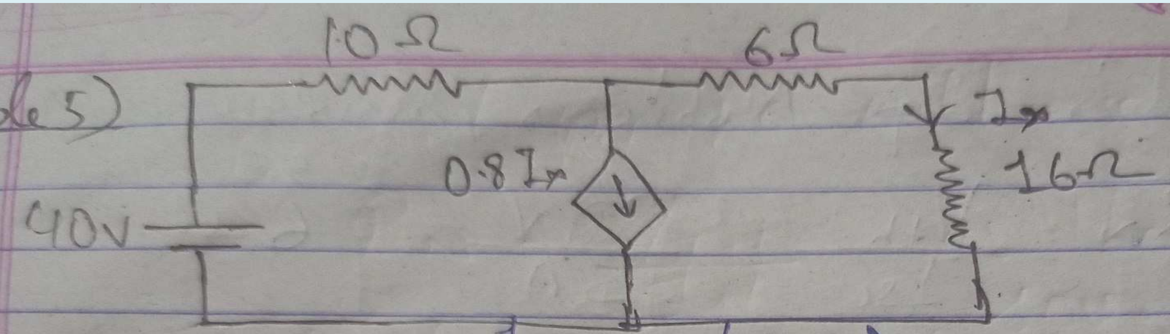
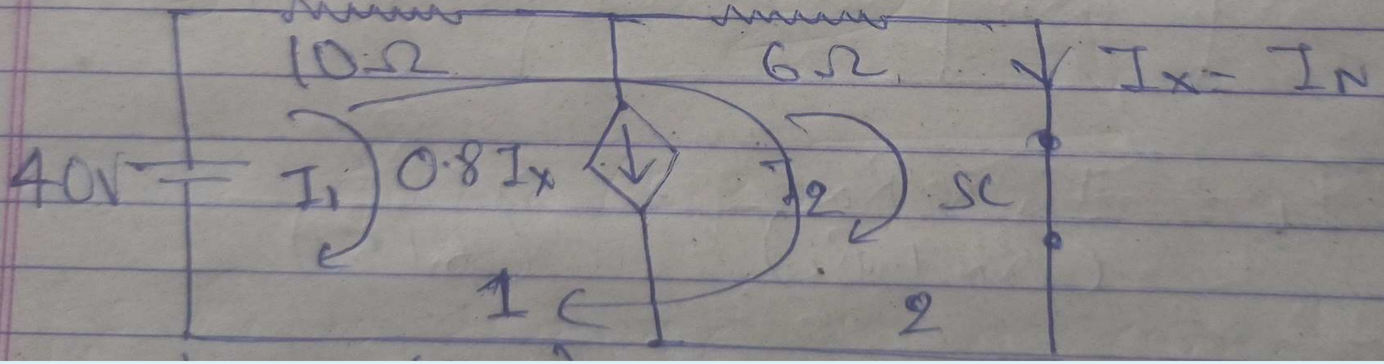


Figure (5-1)

Find the current through 16Ω resistor using Norton's theorem.

Soln:

To find I_N we draw the ckt by removing the 16Ω resistor by short circuit.



Circuit Theorem

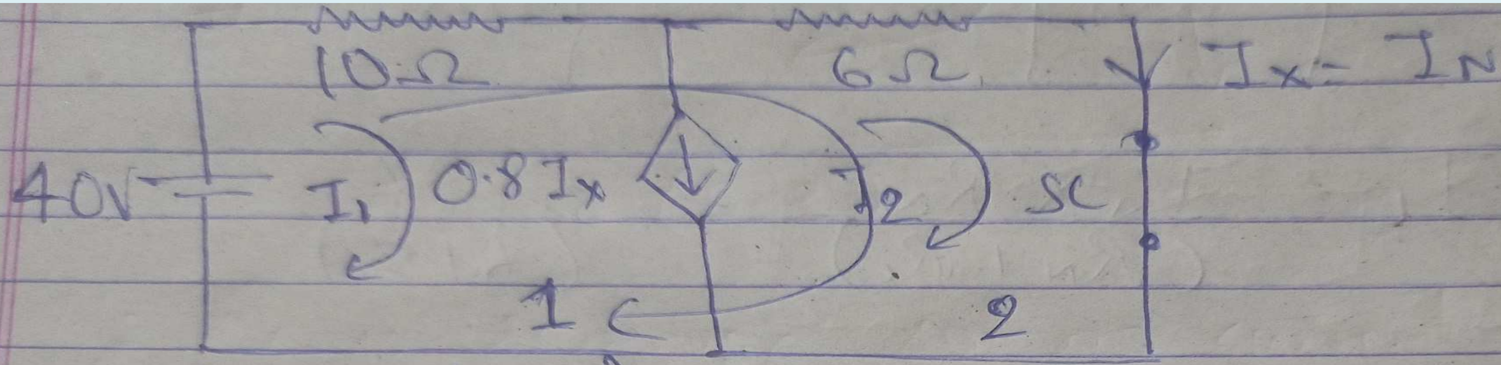


Figure (5.2)

Here $I_N = I_x = I_2$ —— (i)

Let I_1 & I_2 are current flowing in loop 1 & 2 respectively.

Applying KVL in above ckt across complete loop by excluding $0.8 I_x$ dependent current source because it is a case of Su[illegible]

Circuit Theorem

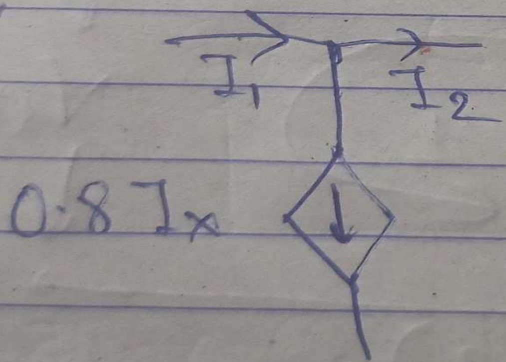
As per KVL in entire loop,

$$10 \times I_1 + 6 \times I_2 - 40 = 0$$

or $$10 I_1 + 6 I_2 = 40$$

$$5 I_1 + 3 I_2 = \cancel{2}20$$

Now as,



Here I_1 is entering & I_2 is leaving and the resultant is $0.8\, I_x$

So as per KCL,

$$I_1 - I_2 = 0.8\, I_x$$

Circuit Theorem

$$I_1 - I_2 = 0.8 I_x$$

$$I_1 = I_2 + 0.8 I_x$$

$$I_1 = 1.8 I_2 \quad [\text{From eqn. (i) } I_2 = I_x]$$

putting value of I_2 in above eqn.

$$5 \times 1.8 I_2 + 3 I_2 = 20$$

$$9 I_2 + 3 I_2 = 20$$

$$12 I_2 = 20$$

$$I_2 = \cancel{\frac{40}{12}} \; \frac{20}{12} \; \frac{5}{3} \; A = I_N$$

So $I_N = \frac{5}{3} A$

Circuit Theorem

Now to find R_N we will use following formula

$$R_N = \frac{V_{TH}}{I_N}$$ as there is dependent source.

To find V_{TH} we will look ~~through the~~ into the ckt after removing R_L, then the ckt diagram is

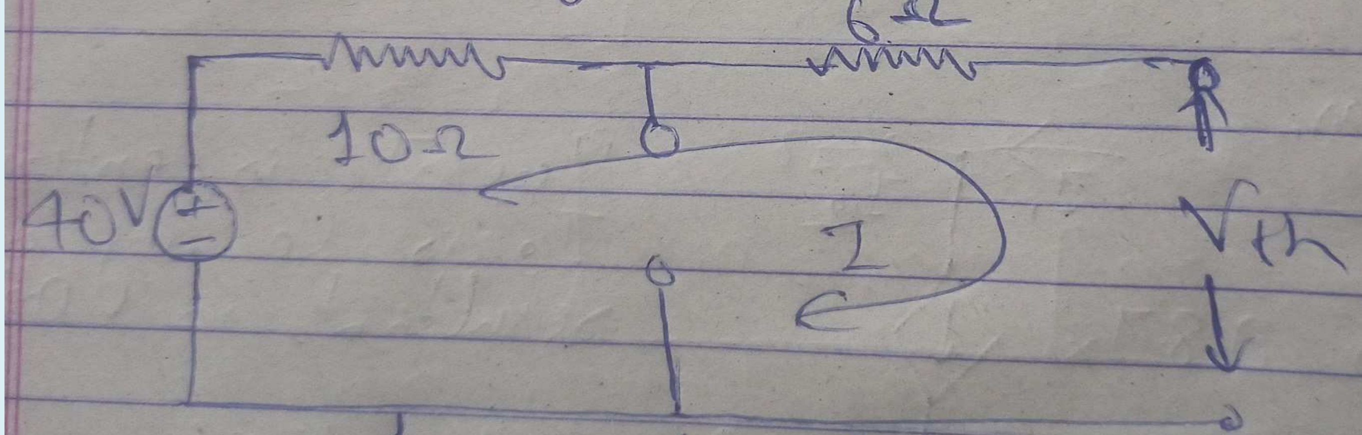


figure (5.3)

Circuit Theorem

As R_L is removed so no current will flow from that arm so $I_x = 0$

Since $I_x = 0$,

Dependent current source $0.8 I_x$ is also 0.

So it is open ckt.

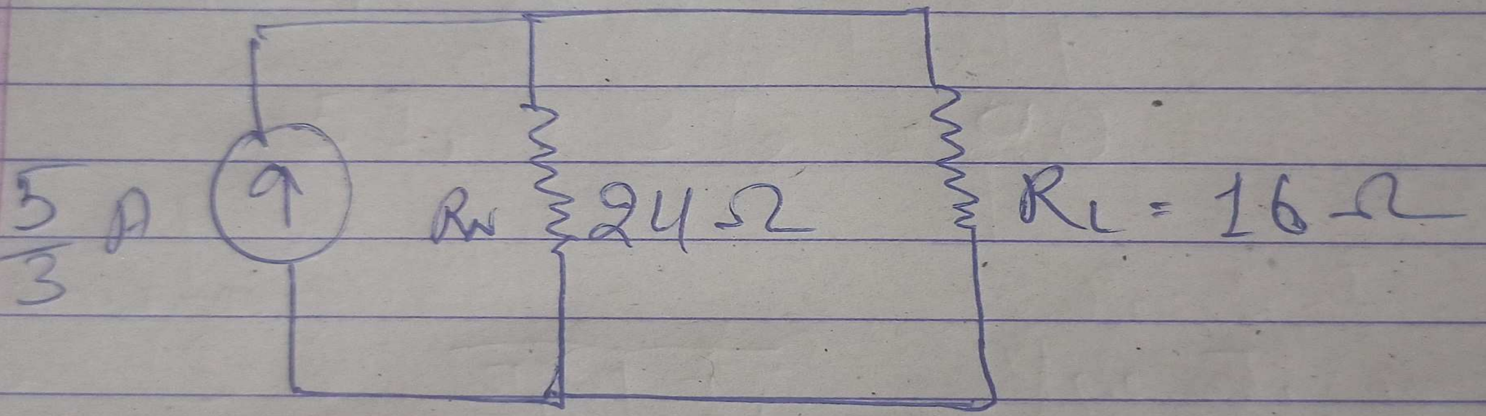
So, $10I + 6I + V_{th} - 40 = 0$

$V_{th} = 40$ V (As the ckt is open so no current will flow in the circuit and hence value of $I = 0$ A

Circuit Theorem

$$R_N = \frac{\cancel{40}\ 8}{(\cancel{5}/3)}\ \Omega = 24\,\Omega$$

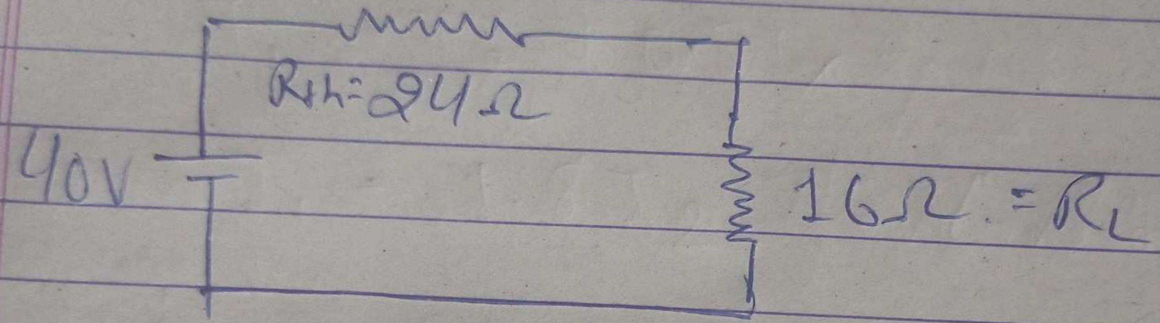
Norton equivalent circuit is,



$$I_N = \frac{\cancel{5}}{\cancel{3}} \times \frac{\cancel{24}\ 8}{\cancel{(24+16)}\ 8} = 1\,A$$

Circuit Theorem

Thevnins equivalent ckt is.



So, $I_L = \frac{40}{(24+16)} = 1A$

So by both theorem current is same through the load. No matter which theorem we apply.

Circuit Theorem

So from these example we can say that a circuit can be analyzed by either Thevenin's theorem or Norton's Theorem. The value of current through and voltage across the load resistor will be same by any of the theorem.

Advantage of Thevenin's theorem or Norton's Theorem over the Superposition theorem is that if the load resistor is varying then we simply replace the load resistor by other load resistor in Thevenin's or Norton's equivalent circuit and find the current and voltage for load resistor. While in case of Superposition we have to analyze entire circuit again if the value of load resistor changes.