TOPIC :- Metallic Reflection
(UG-part-II)

Let us consider a metallic plate of permittivity ϵ_2, magnetic permeability μ_2 and electric conductivity σ_2 be placed above air (i.e non Conducting medium) of electric permittivity ϵ_1 and magnetic permeability μ_1 as shown in fig (1)

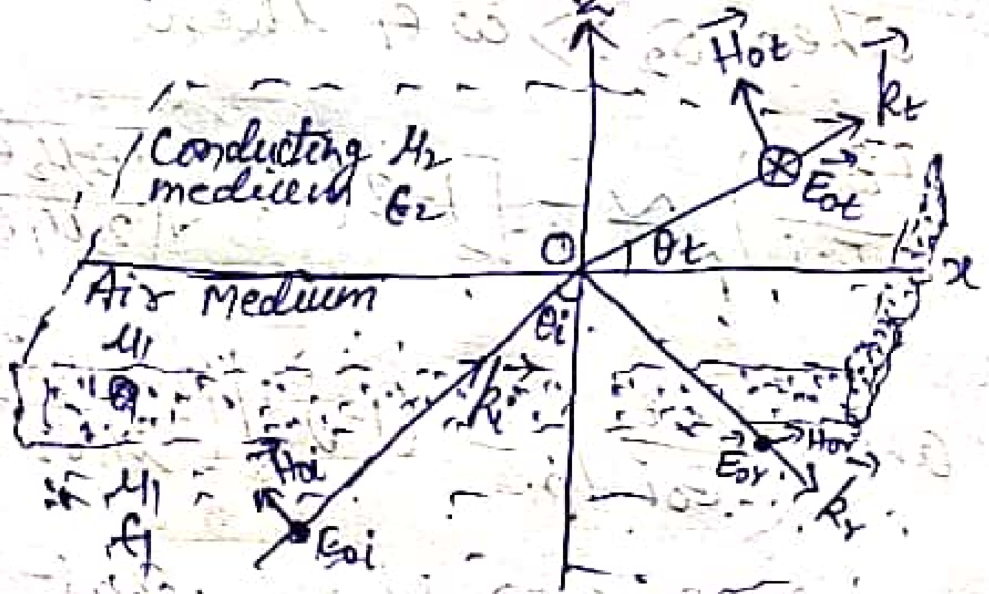

fig (1)

There arises two cases:

Case I :- When $\vec{E}$ vector is perpendicular to the plane of incidence. In this case on applying the continuity of the tangent component of electric field and magnetic field intensity at interface, we have

$$E_{oi} + E_{or} = E_{ot} \longrightarrow (1)$$

and $H_{oi}\cos\theta_i - H_{or}\cos\theta_r = H_{ot}\cos\theta_t \rightarrow (2)$

$\because$ $H_{oi} = \sqrt{\frac{\epsilon_1}{\mu_1}}\, E_{oi}$, $H_{or} = \sqrt{\frac{\epsilon_1}{\mu_1}}\, E_{or}$

and $H_{ot} = \sqrt{\frac{\epsilon_2}{\mu_2}}\, E_{ot} = \frac{k_2}{\omega\mu_2} E_{ot}$; Also $\theta_i = \theta_r$

Hence eq^n (2) becomes,

$$\sqrt{\frac{\epsilon_1}{\mu_1}}\, E_{oi}\cos\theta_i - \sqrt{\frac{\epsilon_1}{\mu_1}}\, E_{or}\cos\theta_i = \frac{k_2}{\omega\mu_2} E_{ot}\cos\theta_t \quad \rightarrow (3)$$

solving eq^n (1) and (3), we have

$$\left(\frac{E_{or}}{E_{oi}}\right)_N = \frac{\cos\theta_i - \frac{k_2}{\omega\mu_2}\sqrt{\frac{\mu_1}{\epsilon_1}}\cos\theta_t}{\cos\theta_i + \frac{k_2}{\omega\mu_2}\sqrt{\frac{\mu_1}{\epsilon_1}}\cos\theta_t} \quad \rightarrow (4)$$

and
$$\left(\frac{E_{ot}}{E_{oi}}\right)_N = \frac{2\cos\theta_i}{\cos\theta_i + \frac{k_2}{\omega\mu_2}\sqrt{\frac{\mu_1}{\epsilon_1}}\cos\theta_t} \quad \rightarrow (5)$$

As for good conductor $\sigma_2 >> \epsilon_2\omega$, $\cos\theta_t = 1$ and $k_2 = \alpha_2 + j\beta_2 = \sqrt{\frac{\mu_2\sigma_2\omega}{2}}\,(1+j)$, equation (4) and (5), becomes

$$\left(\frac{E_{or}}{E_{oi}}\right)_N = \frac{\cos\theta_i - \frac{1}{\omega\mu_2}\sqrt{\frac{\mu_2\sigma_2\omega}{2}}(1+j)\cdot\sqrt{\frac{\mu_1}{\epsilon_1}}}{\cos\theta_i + \frac{1}{\omega\mu_2}\sqrt{\frac{\mu_2\sigma_2\omega}{2}}(1+j)\cdot\sqrt{\frac{\mu_1}{\epsilon_1}}}$$

$$= \frac{\cos\theta_i - (1+j)\sqrt{\frac{\mu_1\sigma_2}{2\omega\mu_2\epsilon_1}}}{\cos\theta_i + (1+j)\sqrt{\frac{\mu_1\sigma_2}{2\omega\mu_2\epsilon_1}}}$$

$$= \frac{\frac{1}{(1+j)}\sqrt{\frac{2\omega\mu_2\epsilon_1}{\mu_1\sigma_2}}\cos\theta_i - 1}{\frac{1}{1+j}\sqrt{\frac{2\omega\mu_2\epsilon_1}{\mu_1\sigma_2}}\cos\theta_i + 1}$$

$$= \frac{-\left[1 - \frac{1}{1+j}\sqrt{\frac{2\omega\mu_2\epsilon_1}{\mu_1\sigma_2}}\cos\theta_i\right]}{\left[1 + \frac{1}{(1+j)}\sqrt{\frac{2\omega\mu_2\epsilon_1}{\mu_1\sigma_2}}\cos\theta_i\right]}$$

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$$= \frac{-\left[1-\frac{(1-j)}{2}\sqrt{\frac{2\omega\mu_2\epsilon_1}{\mu_1\sigma_2}}\cos\theta_i\right]}{1+\frac{(1-j)}{2}\sqrt{\frac{2\omega\mu_2\epsilon_1}{\mu_1\sigma_2}}\cos\theta_i} \qquad \because \frac{1}{1+j} = \frac{1-j}{1-j^2} = \frac{1}{2}(1-j)$$

$$= \frac{-\left[1-(1-j)\sqrt{\frac{\omega\mu_2\epsilon_1}{2\mu_1\sigma_2}}\cos\theta_i\right]}{1+(1-j)\sqrt{\frac{\omega\mu_2\epsilon_1}{2\mu_1\sigma_2}}\cos\theta_i}$$

$$= -\left[1-(1-j)\sqrt{\frac{\omega\mu_2\epsilon_1}{2\mu_1\sigma_2}}\cos\theta_i\right]\left[1+(1-j)\sqrt{\frac{\omega\mu_2\epsilon_1}{2\mu_1\sigma_2}}\cos\theta_i\right]^{-1}$$

$$= -\left[1-(1-j)\sqrt{\frac{\omega\mu_2\epsilon_1}{2\mu_1\sigma_2}}\cos\theta_i\right]\left[1-(1-j)\sqrt{\frac{\omega\mu_2\epsilon_1}{2\mu_1\sigma_2}}\cos\theta_i\right]$$

$$= -\left[1-(1-j)\sqrt{\frac{\omega\mu_2\epsilon_1}{2\mu_1\sigma_2}}\cos\theta_i\right]^2$$

$$= -\left[1-2(1-j)\sqrt{\frac{\omega\mu_2\epsilon_1}{2\mu_1\sigma_2}}\cos\theta_i\right]$$

$$= -\left[1-(1-j)\sqrt{\frac{2\omega\mu_2\epsilon_1}{\mu_1\sigma_2}}\cos\theta_i\right]$$

$\simeq -1$ for $\sigma_2 \to \infty$ i.e for infinite Conductivity $\longrightarrow (6)$

Similarly, from eq^n(5), we have

$$\left(\frac{E_{ot}}{E_{oi}}\right)_N = \frac{2\cos\theta_i}{(1+j)}\sqrt{\frac{2\epsilon_1\mu_2\omega}{\mu_1\sigma_2}}\left[1-(1-j)\sqrt{\frac{\mu_2\epsilon_1\omega}{2\mu_1\sigma_2}}\cos\theta_i\right]$$

$\simeq 0$ for infinite Conductivity $\longrightarrow (7)$

Eq^n (6) and (7) show that the reflected wave ~~and the transmitted wave~~ is approximately π radian out of phase and the transmitted wave is in phase with the incident wave.