RELATION BETWEEN THE AMPLITUDES OF INCIDENT, REFLECTED AND TRANS-MITTED WAVES:-

UG-II-part

Actually the relation between the amplitudes of the reflected and transmitted waves with that of incident wave are known as "Fresnel formulae". In a plane electromagnetic wave the electric vector $\vec{E}$ and magnetic field vector $\vec{H}$ ~~are always~~ are always perpendicular to each other and also to the direction of propagation. The $\vec{E}$ vector is known as light vector so we consider here only about $\vec{E}$ vector. The $\vec{E}$ vector of the incident wave can be oriented in any direction perpendicular to vector $\hat{n}_i$ (i.e ~~a~~ unit vector along the direction of propagation.

For convenience, we consider the following two cases.

Case I:- When the incident wave is polarized such that its $\vec{E}$ vector is ~~parallel~~ perpendicular to the plane of incidence

Case II:- When the incident wave is polarized such that its $\vec{E}$ vector is parallel to the plane of incidence.

Case I:- Incident wave polarized with its $\vec{E}$ vector normal to the plane of incidence.

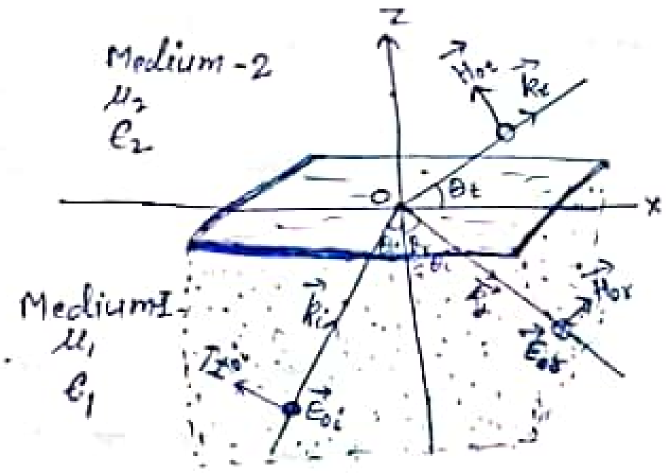

Fig (1)

The electric and magnetic field vector $\vec{E}$ and $\vec{H}$ of the incident wave are perpendicular to the direction of propagation $\vec{k}_i$ as shown in (1). Since the media are isotropic, the electric field vectors of the reflected and transmitted waves will also be normal to the plane of incidence.

Applying boundary condition of the tangential component of the electric field intensity $\vec{E}$ and magnetic field intensity $\vec{H}$ at the interface we have

P.T.O.

$$E_{0i} = E_{0t} + (-E_{0r})$$ (Negative Sign due to phase inversal.)

$\therefore$ $E_{0i} + E_{0r} = E_{0t} \longrightarrow (1)$

and $H_{0i}\cos\theta_i - H_{0r}\cos\theta_i = H_{0t}\cos\theta_t \longrightarrow (2)$ $\because \theta_i = \theta_r$

We know that

$$\vec{H_i} = \frac{\vec{k_i} \times \vec{E_i}}{\mu_1 \omega_i} = \frac{k_i \hat{n}_i \times \vec{E_i}}{\mu_1 \omega_i}$$

$$= \frac{[\sqrt{(\epsilon_1 \mu_1)} \times \hat{n}_i \times \vec{E_i}]}{\mu_1} \quad \left(\because \frac{k_i}{\omega_i} = \sqrt{\mu_1 \epsilon_1}\right)$$

Hence $H_{0i} = \frac{\sqrt{\mu_1 \epsilon_1}}{\mu_1} E_{0i}$ & $H_{0r} = \frac{\sqrt{\epsilon_1 \mu_1}}{\mu_1} E_{0r}$

$= \sqrt{\frac{\epsilon_1}{\mu_1}} E_{0i}$ & $H_{0r} = \sqrt{\frac{\epsilon_1}{\mu_1}} E_{0r}$

Hence eq^n (2) becomes

$$\sqrt{\frac{\epsilon_1}{\mu_1}} E_{0i}\cos\theta_i - \sqrt{\frac{\epsilon_1}{\mu_1}} E_{0r}\cos\theta_i = E_{0t}\sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta_t \longrightarrow (3)$$

Substituting the value of E_{0t} from eq^n(1) in eq^n (3), we have

$$\sqrt{\frac{\epsilon_1}{\mu_1}} E_{0i}\cos\theta_i - \sqrt{\frac{\epsilon_1}{\mu_1}} E_{0r}\cos\theta_i = \sqrt{\frac{\epsilon_2}{\mu_2}}(E_{0i}+E_{0r})\cos\theta_t$$

or, $\sqrt{\frac{\epsilon_1}{\mu_1}} E_{0i}\cos\theta_i - \sqrt{\frac{\epsilon_2}{\mu_2}} E_{0i}\cos\theta_t$

$$= \sqrt{\frac{\epsilon_2}{\mu_2}} E_{0r}\cos\theta_t + \sqrt{\frac{\epsilon_1}{\mu_1}} E_{0r}\cos\theta_i$$

$$\left(\frac{E_{0i}}{E_{0r}}\right)_N = \frac{\sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta_t + \sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_i}{\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_i - \sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta_t}$$

or, $$\left(\frac{E_{0r}}{E_{0i}}\right)_N = \frac{\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_i - \sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta_t}{\sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta_t + \sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_i} \longrightarrow (4)$$

The equation gives the relative electric field amplitude of reflected wave w.r.to incident wave.

Similarly, Substituting the value of E_{0r} from eq^n(1) in eq^n (3), we have the relation of the amplitude of transmitted wave with that of incident wave as

$$\left(\frac{E_{0t}}{E_{0i}}\right)_N = \frac{2\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_i}{\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_i + \sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta_t} \longrightarrow (5)$$

Eq^n (4) and (5) are known as "Fresnel's Equation." For non conducting medium $\mu_1 = \mu_2 = \mu_0$. Hence eq^n(4) and (5) becomes

$$\left(\frac{E_{0r}}{E_{0i}}\right)_N = \frac{\sqrt{\epsilon_1}\cos\theta_i - \sqrt{\epsilon_2}\cos\theta_t}{\sqrt{\epsilon_2}\cos\theta_t + \sqrt{\epsilon_1}\cos\theta_i}$$

$$= \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_2\cos\theta_t + n_1\cos\theta_i} \; ;(\text{as } n \propto \sqrt{\epsilon}) \longrightarrow (6)$$

and $$\left(\frac{E_{0t}}{E_{0i}}\right)_N = \frac{2\sqrt{\epsilon_1}\cos\theta_i}{\sqrt{\epsilon_1}\cos\theta_i + \sqrt{\epsilon_2}\cos\theta_t}$$

$$= \frac{2n_1\cos\theta_i}{n_1\cos\theta_i + n_2\cos\theta_t} \longrightarrow (7)$$

From Snell's law we may put $n_2 = n_1\frac{\sin\theta_i}{\sin\theta_t}$ and find that $\left(\frac{E_{0r}}{E_{0i}}\right)_N = \frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$ and $\left(\frac{E_{0t}}{E_{0i}}\right)_N = \frac{2\cos\theta_i\sin\theta_t}{\sin(\theta_i+\theta_t)}$ $\longrightarrow (8)$

→ Then when $\theta_t > \theta_i$ i.e $\cos\theta_i > \cos\theta_t$ then $\frac{E_{0r}}{E_{0i}}$ = +ve, this shows that the reflecting wave is in phase with incident wave at the interface.

→ when $\theta_t < \theta_i$ i.e $\cos\theta_i < \cos\theta_t$ and then $\frac{E_{0r}}{E_{0i}}$ = −ve, this shows that the reflected wave is π radian out of-phase with incident wave.

these cases are shown in fig (2).

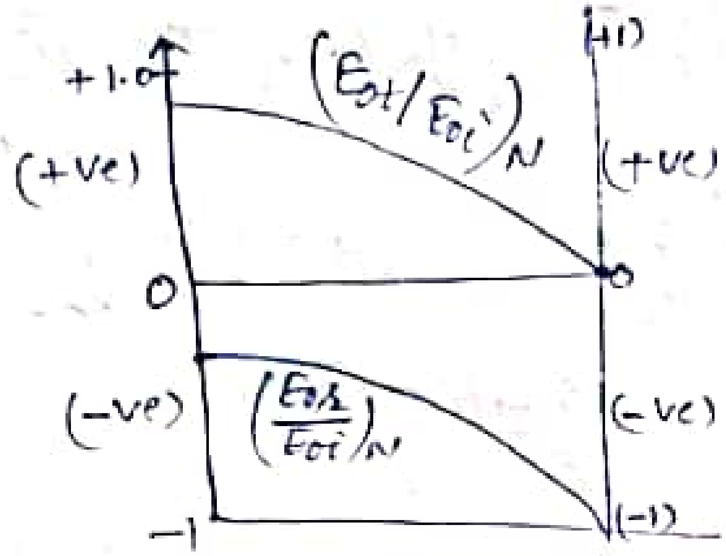

Fig (2)