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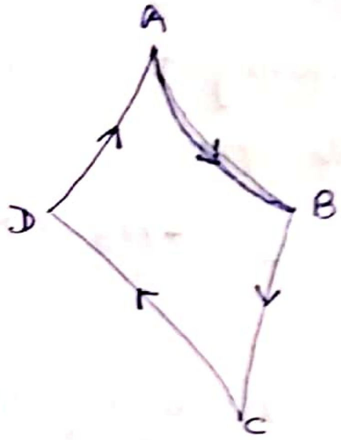
Carnot cycle & its efficiency :-

Carnot employed a reversible cycle to demonstrate the maximum convertibility of heat into work. The system consists of one mole of an ideal gas which is subjected to a series of four successive operations, commonly termed as four strokes as given below:



(I). Isothermal Expansion :
(from A to B)

Volume changes isothermally from V_1 to V_2 at temperature T_2.

net work done –

$$w = P.dv$$

$$w_1 = \int_{V_1}^{V_2} P dv = RT_2 \int_{V_1}^{V_2} \frac{dv}{V}$$

$$w_1 = RT_2 \ln \frac{V_2}{V_1}$$

since, isothermal expansion,

$$dE = 0$$

$$q_1 = w_1 = RT_2 \ln \frac{V_2}{V_1}.$$

(II). Adiabatic expansion :

Volume increases from V_2 to V_3 but in adiabatic process, temperature falls from T_2 to T_1.

$$W_2 = -\int_{T_2}^{T_1} C_V dT$$

$$W_2 = C_V (T_2 - T_1)$$

$q = dE + w$
$0 = dE + w$
$-dE = w$
$w = -C_V dT$

(III) For Isothermal Compression :-

Volume changes from V_3 to V_4. Temperature remains constant.

$$W_3 = RT_1 \int_{V_3}^{V_4} \frac{dv}{v}$$

$$W_3 = RT_1 \ln \frac{V_4}{V_3}$$

during this compression, system looses T_2 K of heat to the system.

$$-q_2 = W_3 = RT_1 \ln \frac{V_4}{V_3}$$

(IV) Adiabatic Compression :-

Final temperature is T_2.

$$W_4 = -\int_{T_1}^{T_2} C_v dT$$

$$W_4 = -C_v (T_2 - T_1)$$

Now, total workdone —

$$W = W_1 + W_2 + W_3 + W_4$$

$$W = RT_2 \ln \frac{V_2}{V_1} + \cancel{C_v (T_2 - T_1)} + RT_1 \ln \frac{V_4}{V_3} - \cancel{C_v (T_2 - T_1)}$$

$$W = RT_2 \ln \frac{V_2}{V_1} + RT_1 \ln \frac{V_4}{V_3}$$

Total amount of heat —

$$q = q_1 - q_2$$

$$q = RT_2 \ln \frac{V_2}{V_1} - RT_1 \ln \frac{V_4}{V_3}$$

(A) Since, $T_2 V_2^{\gamma-1} = T_1 V_3^{\gamma-1}$ —— (1)

(B) $T_2 V_1^{\gamma-1} = T_1 V_4^{\gamma-1}$ —— (2)

dividing eqⁿ — (1) by — (II)

$$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

$$W = R T_2 \ln \frac{V_2}{V_1} - R T_1 \ln \frac{V_2}{V_1}$$

$$W = R \ln \frac{V_2}{V_1} (T_2 - T_1)$$

This is the total work done equal to net heat absorbed is fully satisfied.

* Efficiency of Engine :-

The efficiency of a Carnot cycle or heat engine is defined as the ratio of the work produced to the quantity of heat absorbed at higher temperature.

$$E = \frac{W}{q_2}$$

or

$$\frac{R (T_2 - T_1) \ln \frac{V_2}{V_1}}{R T_2 \ln \frac{V_2}{V_1}}$$

$$E = \frac{T_2 - T}{T_2}$$

for a cyclic process, $q_2 - q_1 = W$

$$E = \frac{q_2 - q_1}{q_2}$$ or $$\boxed{E = \frac{T_2 - T_1}{T_2} = \frac{q_2 - q_1}{q_2}}$$