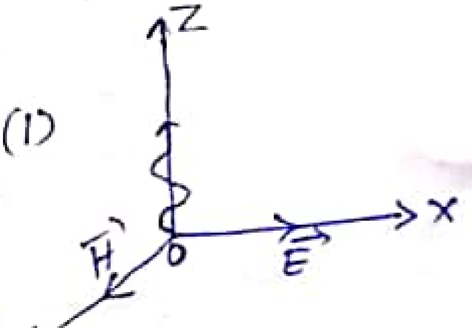
Topic:- PROPAGATION OF PLANE EMW IN CONDUCTING MEDIA

U.G - II

Let us consider the plane polarized electromagnetic wave propagating in z-direction with E in x direction so that

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0 \quad \text{and} \quad \left.\begin{aligned}\vec{E} &= \vec{i}E_x \\ \vec{H} &= \vec{j}H_y\end{aligned}\right\} \quad (1)$$



Fig(i)

$\therefore \vec{\nabla}\cdot\vec{E} = \rho/\epsilon$ gives us

$$\left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)\cdot\left(\vec{i}E_x + \vec{j}E_y + \vec{k}E_z\right) = \rho/\epsilon$$

or, $$\left(\vec{k}\frac{\partial}{\partial z}\right)\cdot\left(\vec{i}\cdot E_x\right) = \rho/\epsilon = 0 \quad \text{--- (2)}$$

Now from Maxwell's equation

$$\vec{\nabla}\cdot\vec{D} = \rho \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t} \quad \text{and} \quad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

we have

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

or, $$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -\mu\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{H}) \quad \text{as } \vec{B} = \mu\vec{H}$$

or, $$\vec{\nabla}(\rho/\epsilon) - \nabla^2\vec{E} = -\mu\frac{\partial}{\partial t}\left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right)$$

$$= -\mu\frac{\partial}{\partial t}\left(\sigma\vec{E} + \epsilon\frac{\partial\vec{E}}{\partial t}\right)$$

as $\vec{D} = \epsilon\vec{E}$ & $\vec{J} = \sigma\vec{E}$

or, $$= -\mu\sigma\frac{\partial\vec{E}}{\partial t} - \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

or, $$\nabla^2\vec{E} = \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} + \mu\sigma\frac{\partial\vec{E}}{\partial t} + \vec{\nabla}(\rho/\epsilon) \quad \text{(3)}$$

Similarly $$\nabla^2\vec{H} = \mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2} + \mu\sigma\frac{\partial\vec{H}}{\partial t}.$$

Now using equation (1) & (2) in eq^n (3), we get the equation for conducting medium as

$$\left.\begin{aligned} \frac{\partial^2 E_x}{\partial z^2} &= \mu\epsilon \frac{\partial^2 E_x}{\partial t^2} + \mu\sigma \frac{\partial E_x}{\partial t} \\ \& \quad \frac{\partial^2 H_y}{\partial z^2} &= \mu\epsilon \frac{\partial^2 H_y}{\partial t^2} + \mu\sigma \frac{\partial H_x}{\partial t} \end{aligned}\right\} (4)$$

Assuming harmonics variation of E_x with respect to t, we may write

$$E_x = E_0 e^{j\omega t} \longrightarrow (5a)$$

so that $\frac{\partial E_x}{\partial t} = (j\omega) E_0 e^{j\omega t} \times E_0 = j\omega E_x \longrightarrow (5a)$

and $\frac{\partial^2 E_x}{\partial t^2} = (j\omega)(j\omega) e^{j\omega t} \times E_0 = -\omega^2 E_0 e^{j\omega t}$

$= -\omega^2 E_x \longrightarrow$ (5b)

putting the value of eq^n (5a) and (5b) in eq (4) we have

$$\frac{\partial^2 E_x}{\partial z^2} + \omega^2 \mu\epsilon E_x - j\omega\sigma\mu E_x = 0 \longrightarrow (6)$$

or, $\frac{\partial^2 E_x}{\partial z^2} - (j\omega\sigma\mu + \omega^2\mu\epsilon) E_x = 0$

Let us put γ, propagation constant, as

$$\gamma^2 = j\omega\sigma\mu + \omega^2\mu\epsilon \longrightarrow (7)$$

then eq^n (6) becomes

$$\frac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0 \longrightarrow (8)$$

which is the wave equation for E, the ele vector, which can be solved by consider

$$E_x = E_0 e^{-\gamma z} \longrightarrow (9)$$

where $\gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\epsilon}$

$= \sqrt{j\omega\mu\sigma\left(1 + j\frac{\omega\epsilon}{\sigma}\right)}$

$= \sqrt{j\omega\mu\sigma}$ (as for good conductor $\frac{\sigma}{\omega\epsilon} >> 1$)

Again let $\gamma = \sqrt{j\omega\mu\sigma} = \alpha + j\beta$

∴ on squaring we get

$\alpha^2 - \beta^2 + 2\alpha\beta j = j\omega\mu\sigma$

Equating real & imaginary part, we have

$\alpha^2 - \beta^2 = 0$ i.e $\alpha = \beta$

as $2\alpha\beta = \omega\mu\sigma$

or $\alpha^2 = \beta^2 = \frac{\omega\mu\sigma}{2}$ $\therefore \alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$

Hence $\gamma = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}}$ ⟶ (10)

putting the value of eqn (10) in equation (9), we obtain,

$$E_x = E_0 \exp\left[-(1+j)\sqrt{\left(\frac{\omega\mu\sigma}{2}\right)}z\right]$$

$$= E_0 \exp\left\{-\sqrt{\left(\frac{\omega\mu\sigma}{2}\right)}\cdot z\right\} \exp\left\{-j\sqrt{\left(\frac{\omega\mu\sigma}{2}\right)}\cdot z\right\} \longrightarrow (11)$$

In equation (11), attenuation factor is

$$\exp\left[-\sqrt{\left(\frac{\omega\mu\sigma}{2}\right)}\cdot z\right]$$

and the phase factor is $\exp\left[-j\sqrt{\left(\frac{\omega\mu\sigma}{2}\right)}\cdot z\right]$

Thus eqn (11) predicts that the field attenuates exponentially and is retarded linearly in phase with increasing z, since σ is very large for good conductor.

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