[C]

TOPIC :- Electromagnetic waves in Free Space

UG. II part.

The fundamental relations required to solve any electromagnetic problem are given below

$$\left.\begin{aligned} \vec{\nabla}\cdot\vec{D} &= \rho \\ \vec{\nabla}\cdot\vec{B} &= 0 \\ \vec{\nabla}\times\vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla}\times\vec{H} &= \vec{J}+\frac{\partial \vec{D}}{\partial t} \end{aligned}\right\} \quad (1)$$

In order to fit these relations so as to be applicable to the different media, we have three more relations as

$$\vec{D}=\epsilon\vec{E}, \quad \vec{B}=\mu\vec{H} \text{ and } \vec{J}=\sigma\vec{E} \longrightarrow (2)$$

where ϵ = permittivity, μ = permeability and σ = conductivity of the medium. For free space, $J=0$, $\rho=0$, $\sigma=0$, $\mu=1$, $\epsilon=1$ $\epsilon_r=1$ So that $\vec{D}=\epsilon_0\vec{E}$ and $\vec{B}=\mu_0\vec{H}$. where ϵ_0 and μ_0 are respectively free space permittivity and permeability.

Obviously for free space eq^n (1) becomes

$$\left.\begin{aligned} \vec{\nabla}\cdot\vec{D} &= 0 \quad \text{---(a)} \\ \vec{\nabla}\cdot\vec{B} &= 0 \quad \text{---(b)} \\ \vec{\nabla}\times\vec{E} &= -\frac{\partial \vec{B}}{\partial t} \quad \text{--(c)} \\ \vec{\nabla}\times\vec{H} &= \frac{\partial \vec{D}}{\partial t} \quad \text{.(d)} \end{aligned}\right\} \quad (3)$$

Taking curl of eq^n(3c), we get

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = -\left(\vec{\nabla}\times\frac{\partial \vec{B}}{\partial t}\right)$$

$$= -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$= -\frac{\partial}{\partial t}(\vec{\nabla}\times\mu_0\vec{H})$$

$$= -\mu_0\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{H})$$

$$= -\mu_0\frac{\partial}{\partial t}\left(\frac{\partial \vec{D}}{\partial t}\right) \text{ from (3d)}$$

$$= -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \longrightarrow (4)$$

further, we know that from $(\vec{A} \times \vec{B} \times \vec{C})$ formula,

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E}$$

$$= \vec{\nabla}\left(\vec{\nabla} \cdot \frac{\epsilon_0 \vec{E}}{\epsilon_0}\right)$$

$$= \frac{1}{\epsilon_0} \vec{\nabla}(\vec{\nabla} \cdot \vec{D}) = 0 \rightarrow (5)$$

from eq^n (4) and (5) we obtain,

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \longrightarrow (6)$$

Similarly, we can obtain

$$\nabla^2 \vec{H} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \longrightarrow (7)$$

If we compare these two equation ~~them~~ with the wave equation in general

$$\frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

then we can say that the field vectors $\vec{E}$ and $\vec{H}$ propagate as wave in free space with velocity

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad \left(\because \mu_0 = 4\pi \times 10^{-7}\ H/m,\ \epsilon_0 = 8.8542 \times 10^{-12}\ F/m\right)$$

$$= 2.9999 \times 10^{8}\ m/s$$

which is equal to velocity of light. Thus light is an electromagnetic wave.