The three different types of velocities which are used to study of gases are :-

1). Most Probable velocity :-

It is defined as the velocity of gas possessed by maximum no. of molecules at a given temperature.

The equation for Maxwell distribution of velocities may be written as —

$$\frac{N}{dU_x} = 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} \exp\left(-\frac{MU_x^2}{2RT}\right) U_x^2 \quad \text{—— (1)}$$

for maxima & minima conditions, This equation is differentiated w.r.t U_x and equated to zero. Then we get —

$$\left(1 - \frac{MU_x^2}{2RT}\right) U_x \exp\left(-\frac{MU_x^2}{2RT}\right) = 0.$$

since, $$U_x \exp -\frac{MU_x^2}{2RT} \neq 0.$$

Then, $$1 - \frac{MU_x^2}{2RT} = 0.$$

or $$\frac{MU_x^2}{2RT} = 1$$

$$MU_x^2 = 2RT$$

$$\boxed{U_x = \sqrt{\frac{2RT}{M}}}$$

where, 'U_x' is the most Probable velocity. M = Mol. mass of the gas.

Problems

(1) Calculate the most probable velocity of H_2 gas.

Given, molar mass of H_2 = $2.016 \ g \ mol^{-1}$
= $2.016 \times 10^{-3} \ kg \ mol^{-1}$.

(Ans. $1.50 \times 10^{3} \ m \ sec^{-1}$.)

(2) The most probable speed (velocity) at T K of CO_2 gas is 9×10^{4} cm sec^{-1}. Calculate the temperature.

(Hint $1 J = 10^{7}$ erg
$R = 8.314 J \ K^{-1} mol^{-1}$)

(Ans → 2143 K)

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2) Average velocity :-

The average velocity is given by the arithmatic mean of the different velocities possessed by the molecules of the gas at a given temperature.

If $u_1, u_2, u_3 \ldots\ldots$ are the velocities possessed by $n_1, n_2, n_3 \ldots\ldots$ number of molecules respectively, then average velocity will be -

$$\langle U_x \rangle = \frac{u_1 + u_2 + u_3 + \ldots\ldots\ldots U_N}{N}$$

$$= \frac{1}{N} \sum_{i=1} U_{x_i}$$

The average velocity may also be defined as the probability that fraction of molecules having velocities between U_x & $U_x + dU_x$, which is represented by -

$$f \langle U_x \rangle dU_x$$

or $$\langle U_x \rangle = \int_0^\infty U_x f \langle U_x \rangle dU_x \quad \text{——— (1)}$$

where, $$f\langle U_x \rangle dU_x = 4\pi \left(\frac{M}{2\pi RT} \right)^{3/2} \exp\left(-\frac{MU_x^2}{2RT} \right) U_x^2 dx$$

putting this value in eq^n -(1) and solving for the intigral we get -

$$\langle U_x \rangle = \left(\frac{8RT}{\pi M} \right)^{1/2}$$

Numericals :-

(1). Calculate the average speed of CO_2 gas at 1684 K temperature?

Hints:- $\pi = 3.141$

$R = 8.314\ J\ K^{-1}\ mol^{-1}$

$(1J = 10^7\ erg)$

$M_{CO_2} = 44\ g\ mol^{-1}$.

(2). The average speed at T K temperature of N_2 gas is $8 \times 10^5\ cm\ sec^{-1}$. calculate the temperature (T).

Hints: $\pi = 3.141$

$R = 8.314\ J\ K^{-1}\ mol^{-1}$

$1J = 10^7\ erg$

$M_{N_2} = 28\ g\ mol^{-1}$.

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(3). Root mean square (RMS) velocity :-

The rms velocity is defined as the square root of the mean of the squares of different velocities possessed by molecules of gas at a given temperature.

The rms velocity is given by—

$$\bar{u} = \left(\frac{1}{N} \sum_{i=1}^{N} u_{x_i}^2 \right)^{1/2}$$

This may also be written as —

$$\bar{u} = \int_0^\infty u_x^2\, f\langle u_x \rangle\, du_x$$

where, $f\langle u_x \rangle du_x$ represents the fraction of molecules having velocities between u_x & $u_x + du_x$.

where,

$$f\langle U_x\rangle dU_x = 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} \exp\left(-\frac{MU_x^2}{2RT}\right) U_x^2 \, dU_x .$$

On substituting this value in intigral & solving we get -

$$\bar{u} = \left(\frac{3RT}{M}\right)^{1/2} .$$

* Relationship between Most Probable velocity (U_x), Average velocity $\langle U_x\rangle$ and rms velocity $\bar{u}$:-

Since,

Most Probable velocity $(U_x) = \left(\frac{2RT}{M}\right)^{1/2}$

Average velocity $\langle U_x\rangle = \left(\frac{8RT}{\pi M}\right)^{1/2}$

and. RMS velocity $\bar{u} = \left(\frac{3RT}{M}\right)^{1/2}$

Therefore, on compairing them,

$$(U_x) = \langle U_x\rangle = \bar{u} .$$

$$\left(\frac{2RT}{M}\right)^{1/2} = \left(\frac{8RT}{\pi M}\right)^{1/2} = \left(\frac{3RT}{M}\right)^{1/2}$$

$$1 = 1.128 = 1.1224$$

Thus,

average velocity $\langle U_x\rangle = 0.9213 \times \bar{u}$

& Most Probable velocity $(U_x) = \sqrt{2/3}\ \bar{u} = 0.816\ \bar{u}$.