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Part - I (B.Sc):

Stationary Waves

— Dr. S. Roy

When two identical waves (either transverse or longitudinal) travel through a medium along the same line in opposite direction, superposition of waves produces "Stationary waves."

When a wave is sent along a string or along the air-column of a pipe, it is reflected at the ~~end~~ other end and superimpose upon the incident wave to produce stationary wave. When such waves are formed then certain particles of the medium remain at rest, while some other particles undergo maximum displacement compared to others. The former are called "nodes" and later the "antinodes."

In order to form a stationary wave following conditions ~~must be~~ must be obeyed.

(i) medium must not be infinite in length.

(ii) It should have boundary.

Suppose a plane progressive wave of amplitude 'a' is travelling with velocity v along x-direction, its equation can be written as

$$y_1 = a \sin \frac{2\pi}{\lambda}(Vt - x) \quad \text{- - - (1)}$$

where y_1 is the displacement at a point x at time t. Let this wave be reflected from rigid boundary at $x = 0$. Then displacement of reflected wave in x-direction can be written as

$$y_2 = a' \sin \frac{2\pi}{\lambda}(Vt + x) \longrightarrow (2)$$

where a' is the amplitude of reflected wave from principle of superposition, the resultant displacement (y) at point x at instant 't' will be

$$y = y_1 + y_2$$

$$= a \sin \frac{2\pi}{\lambda}(vt - x) + a' \sin \frac{2\pi}{\lambda}(vt + x) \longrightarrow (3)$$

The boundary condition is that at $x = 0$, $y = 0$. So eq^n (3) gives us,

$$0 = a \sin \frac{2\pi}{\lambda}(vt) + a' \sin \frac{2\pi}{\lambda}(vt)$$

$$\therefore a' = -a \longrightarrow (4)$$

Hence eq^n (3) becomes,

$$y = a\left[\sin \frac{2\pi}{\lambda}(vt - x) - \sin \frac{2\pi}{\lambda}(vt + x)\right]$$

$$= -2a \sin \frac{2\pi x}{\lambda} \cos \frac{2\pi}{\lambda} vt \longrightarrow (5)$$

Eq^n (5) is the equation of stationary waves with same period as the original waves but of amplitude $-2a \sin \frac{2\pi x}{\lambda}$. Thus the resultant displacement changes with position as well as with time (t).

At point where $\frac{2\pi x}{\lambda} = 0, \pi, 2\pi, \cdots$

or $x = 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \cdots$

We have $\sin \frac{2\pi x}{\lambda} = 0$. This gives $y = 0$ i.e at all these points, the displacement are always zero and is known as "nodes".

Similarly at the points where

$$\frac{2\pi x}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \cdots$$

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \cdots$$

We have $\sin\frac{2\pi x}{\lambda} = \pm 1$ this gives $y =$ max. i.e at these points, the displacement of the Stationary waves becomes maximum & are known as "Antinodes". - - - - - - - - - - -