PG- IVth Sem)

Dr. S. Roy

Mobility of charged particle in plasma

In an external electric field, the charged particles of a plasma acquire a directed motion or drift motion. The drift velocity is proportional to the aplied field

i.e Drift velocity $\propto$ Applied electric field

or Drift velocity $= \mu \times$ Applied electric field

$$\therefore \mu = \frac{\text{Drift velocity}}{\text{Applied electric field}} = \frac{v_d}{E}$$

where μ is the constant of proportionality and is called "mobility". Thus the drift velocity acquired by charged particle per unit electric field is known as "mobility". Mobility of electrons is much greater than that of ions i.e $\mu_e \gg \mu_i$. Therefore the mobility of ions in plasma may be neglected.

Let us suppose that electrons under an alternating electric field of magnitude E_0 and frequency ω in the direction of x-axis is moving with a velocity whose components are v_x and v_y along the x-axis and y-axis respectively. Let a transverse magnetic field $\vec{H}$ along the z-axis be applied. Then the equation of motion of the electron are

$$\left.\begin{aligned} m\frac{dv_x}{dt} + m\nu_c v_x + Hev_y &= eE_0 e^{j\omega t} \\ \text{or} \quad \frac{dv_x}{dt} + \nu_c v_x + \omega_H v_y &= \frac{eE_0}{m} e^{j\omega t} \end{aligned}\right\} \cdots (1)$$

And, $m\frac{dv_y}{dt} + m\nu_c v_y - Hev_x = 0$

or $$\frac{dv_y}{dt} + v_c v_y - \omega_H v_x = 0 \longrightarrow (2)$$

where, v_c = the restoring force per unit velocity per unit mass & is known as the collision frequency of the electron with gas molecules. $\omega_H = \frac{eH}{m}$ & is known as electron cyclotron frequency.

To solve eqn (1) and (2), it is assumed that

$$v_x = Ae^{j\omega t} \text{ and } v_y = Be^{j\omega t} \longrightarrow (2a)$$

$$\therefore \frac{dv_x}{dt} = Aj\omega e^{j\omega t} \text{ \& } \frac{dv_y}{dt} = Bj\omega e^{j\omega t}$$

putting these value in eqn (1) and (2), we obtain first from eqn (2)

$$\cancel{Bj\omega e^{j\omega t} + v_c B}$$

$$B[j\omega + v_c] = A\omega_H$$

$$\therefore B = A\frac{\omega_H}{[j\omega + v_c]} \longrightarrow (3)$$

$$\therefore v_y = Be^{j\omega t} = A\frac{\omega_H}{[j\omega + v_c]}e^{j\omega t}$$

$$= \frac{\omega_H}{[j\omega + v_c]} v_x \longrightarrow (4)$$

Secondly from eqn (1), we have

$$A\left[v_c + j\omega + \frac{\omega_H^2}{(v_c + j\omega)}\right] = \frac{e}{m}\cdot E_0$$

$$\therefore A = \frac{\frac{e}{m}\cdot E_0}{\left[v_c + j\omega + \frac{\omega_H^2}{(v_c + j\omega)}\right]} \longrightarrow (5)$$

P.G.(IV) Mobility

- Dr. S. Roy

From eq^n (2a) and (5), we have

$$v_x = \frac{\frac{e}{m} E_0 e^{j\omega t}}{(\nu_c + j\omega)^2 + \omega_H^2} (\nu_c + j\omega) \longrightarrow (6)$$

For a d.c. field, $\omega = 0$. Hence we get

$$v_x = \frac{\frac{e}{m} \cdot E_0 \nu_c}{\nu_c^2 + \omega_H^2}$$

or $$\frac{v_x}{E_0} = \frac{e\nu_c}{m(\nu_c^2 + \omega_H^2)}$$

or $$\mu_H = \frac{e\nu_c/m}{\nu_c^2\left[1 + \frac{\omega_H^2}{\nu_c^2}\right]}$$ (From definition of Mobility)

$$= \frac{e/m}{\nu_c[1 + \omega_H^2 \tau^2]}$$

Where, $\tau = \frac{1}{\nu_c}$, the time between successive collisions between electron & gas mol.

$$= \frac{(e/m)}{\nu_c\left[1 + \frac{e^2H^2}{m^2} \cdot \frac{\lambda_e^2}{v_r^2}\right]}$$

$\because \omega_H = \frac{eH}{m}$

& $\tau = \frac{v_r}{\lambda_e}$

Also, $\lambda_e = L/P$

L = Mean free path of the electron at a pressure of 1 mm of Hg.

$$= \frac{\mu}{1 + \frac{e^2L^2}{m^2 v_r^2} \cdot \frac{H^2}{P^2}}$$

$$\therefore \mu_H = \frac{\mu}{1 + C_1 \frac{H^2}{P^2}} \longrightarrow (7)$$

where $\mu = \frac{e}{m\nu_c}$ & $C_1 = \frac{e^2L^2}{m^2 v_r^2}$

P.G.(IV) Mobility

- Dr. S. Roy

v_r = Random velocity of the electron.

In equation (7), (H/P) is called Reduced magnetic field. If the mobility of the electron is determined at various values of (H/P) then the variation of $\frac{\mu}{\mu_H}$ against H^2/p^2 should be a straight line and the slope of the curve should provide the value of C_1.